Response of an offshore wind turbine to simultaneous wind and wave loads

\[(EIv'')'' = q - \rho A\ddot{v}\]

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Preface

This is the report of a 35 ECTS master thesis written in the Mechanical Engineering Department in DTU as part of M.Sc. studies in Wind Energy. The project was held partly in collaboration with Danish Hydraulic Institute. During the thesis, efforts have been spent on the use of MIKE 21 SW software for wave modeling and Matlab programming for aeroelastic modeling.

I would like to thank my supervisors Jens Nørkaer Sørensen and Henrik Kofoed Hansen and my co-supervisor Henrik Bredmøse for their time spent to discuss with me about my thesis subject and their help by sharing their knowledge and experience with me. I would also like to thank Patrick Dich Grode for his help and guidance on the DHI related part of my thesis.

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Abstract

As offshore wind power technology gains more and more importance, more challenges arise related with offshore wind turbine installations. One of the most important challenges faced by offshore wind industry is the reliable prediction of loads on offshore wind turbine support structures for safe and optimized design purposes.

Unlike onshore wind turbine design cases where site-independent choice of turbine classes can be made for a certain wind speed range, offshore wind turbine design requires site-specific analysis as it depends on many site-related external conditions such as wave height, water depth and soil types. Therefore, an assessment of the external conditions at the site becomes necessary for the structural components of the offshore wind turbine, especially for the support structure in order to be able to describe the load conditions for design purposes in a realistic manner. For this purpose efforts have been made in developing aeroelastic codes that simulates the response of the wind turbine under both aerodynamic and hydrodynamic loading.

An important area of interest where such an analysis is of importance is the North Sea region where a large number of installed, ongoing and planned offshore wind turbine installations exist. MIKE 21 SW is one of the state of the art wave modeling software developed by Danish Hydraulic Institute (DHI). Ports and offshore department of DHI has existing models that simulate wave conditions in the United Kingdom and North Sea region. It is of interest of DHI to combine their wave model results with an offshore aeroelastic code for a simulation of load case definitions on an offshore wind turbine.

This report addresses to the assessment of wave conditions at a deep water site location at German Bight area in North Sea and implementation of a simple aeroelastic code for offshore wind turbine for extreme load case and response definitions under simultaneous wind and wave loading at that location.
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Introduction

Global Energy Demand and Wind Power

Global demand for energy is increasing at a breath taking rate. It is stated by Global Wind Energy Council (GWEC) that "Depending on the efficiency measures implemented, by 2030 world energy needs are predicated to be between 30 and 60% higher than current levels. The IEA estimates that around 4,500 GW of new energy capacity needs to be installed before 2030, requiring investments of more than US$ 13 trillion." As the energy demand increases globally, supplies for fossil fuel, which are primary sources of power generation in the world, are becoming rare and more difficult to extract and therefore becoming more expensive. Besides the limitations of fossil fuel sources against increasing energy demand, another concern that our world is facing today regarding the use of fossil fuels is the climate change due to CO2 emissions. As pronounced by Intergovernmental Panel on Climate Change (IPCC) in their 4th Assessment Report released in 2007, "Long-lived greenhouse gases (LLGHGs), for example, CO2, methane (CH4) and nitrous oxide (N2O), are chemically stable and persist in the atmosphere over time scales of a decade to centuries or longer, so that their emission has a long-term influence on climate." [1]

The oil crisis in 1970’s had already led to seeking and development of alternative energy sources which are renewable, clean and feasible. Now, together with the awareness of climate change being higher than ever, renewable energy sources are seen as important solutions which will help meeting the worlds increasing energy demand as well as reducing the greenhouse emissions.

Among the renewable energy sources which are deployed around the world such as solar, wind, biomass, thermal and biofuel; wind power developed into one of the important players in world’s energy markets as they kept improving in their efficiency, power rating and reliability. According to GWEC facts, over the past ten years until 2008, the global wind capacity continued growing at an average cumulative rate around 30%. [2]

Offshore Wind Power

Wind energy market is mainly dominated by onshore applications and despite the fact that wind power is a fast growing market, it faces a number of challenges in onshore
installations. These include, for example, wind speed limitations, noise emissions, use of land permissions and legislations. The continuous development in wind turbine technology to overcome these challenges and to exploit more power from the wind resulted in development of offshore wind turbine technologies. Compared to onshore wind energy, the advantages of offshore wind farms are several. They are located at a sufficient distance, therefore noise emissions and visual impact to the surroundings is not a problem. Moreover, the wind patterns in sea are more uniform and wind speeds are higher which favors the use of larger wind turbines and enables higher power extraction from wind. There are also fewer parties concerned regarding the legal issues on site location. Furthermore, it is shown by environmental analysis that offshore wind farms have no considerable negative impact on marine life. Due to these advantages, offshore wind energy has large potential to help meeting worldwide energy needs with much of the current installations concentrated in North Western Europe, mainly Denmark, the United Kingdom, the Netherlands, Sweden and Germany.

Further advantages of offshore wind power is found when the focus is European energy market, as in 2007 EU Heads of State and Government set a series demanding climate and energy targets to be met by 2020. These targets, which are also known as "20-20-20" targets, aim at reducing the EU greenhouse emissions to at least 20% below 1990 levels and providing 20% of EU energy consumption from renewable energy sources. It is seen that offshore wind energy can become a reliable and competitive energy source and can play an important role in meeting these targets, considering the ongoing progress in offshore wind turbine technologies so far.

Apart from the advantages of offshore wind energy mentioned so far, there are also challenges that the offshore community has to face when developing offshore technologies and expanding its market. The main issues related with offshore wind turbine installations are connected with the cost-effectiveness. Despite its advantages and higher potential for large scale mega-turbines and large investments, offshore wind turbines are relatively more challenging and therefore more costly in the aspects of installation, operation and maintenance, grid connecting, and support structure when compared to onshore turbines. Among these challenges, this master project falls in the category of the structural challenges which are related with the development of cost-effective support structures which consist of foundation and tower. One of the biggest problems foundations are facing is the scour process. Different types of seabed and marine conditions have to be tested in detail, in order to make improvements in the foundations since it is an important issue that puts turbines in danger.

Another very important challenge in the design of offshore turbine support structures is the proper prediction of fatigue and extreme loads, which are only possible with aero-elastic simulation codes that can be used to predict the dynamic behavior of the entire system under simultaneous aerodynamic and hydrodynamic loading. This problem on which this project focuses will be explained in the following section.
Aero-elastic codes for offshore wind turbines

For onshore wind turbines, a type certification where a type class for the generic wind condition is assigned for the turbines chosen for the site is adequate for load case definitions. For offshore wind turbines on the other hand, external conditions are varying significantly depending on water depth, soil conditions and wave heights. Therefore, the definition of turbine class in terms of wind speed and turbulence intensity can only be applied to the offshore turbine’s top parts (machinery and rotor blades) provided that no resonance effects occur due to wave loading. However, for the support structure (tower and foundation) the environmental conditions which are characteristic at the site have to be taken into account which makes the use of a generic approach, as in the onshore case, not possible. Therefore, the design of the support structure has to be site specific and detailed knowledge of site conditions such as water depth, soil conditions, extreme wind and wave data are necessary.[6]

The focus on the site assessment for structural design of offshore wind turbine structure is getting description of the combined distributions of wind and waves and if possible their directional relations, e.g. misalignment should be included. In order to be able to describe the load conditions for design of the support structures, the structural response of the turbine under the effect of wind and waves have to be analyzed in a realistic manner. For this purpose, efforts have been made in developing aero-elastic codes which simulates the structural response of the turbine components subjected both aerodynamic and hydrodynamic loading.[6]

Today only a small number of codes exist for simulating the dynamic behavior and loads of offshore wind turbines which are under the effect of combined wind and wave loading. As there are no measurements available and only little practical experience exists, the verification and development of the codes can be made by a benchmark test. For this purpose, under the coordination of National Renewable Energy Laboratory (NREL), a collaboration called "Offshore Code Comparison Collaboration" (OC3) was established in 2005. Project objectives mainly included investigating the reliability and accuracy of the simulation results and identification of code capabilities and limitations of the theories implemented. Almost all of the existing simulation codes for offshore wind turbines took part in the project and formed an important database with all the code results for a number of standardized cases which provides a platform for the verification of individual work on the same topic.[5]

Purpose of the Work

Connected with the definition of the problem in the previous section, the purpose of this thesis is to investigate combined wind and wave loads on an offshore wind turbine at a deep water location in North Sea by means of

• a wave model set up using the commercial wave modeling software MIKE21 SW for wave condition analysis
• a simple aero-elastic model developed on Matlab that uses the outputs of the wave analysis as wave input to calculate the dynamic structural response of the turbine under combined wind and wave loading

The investigation of combined wind and wave loads include

• Comparison of the code with the test cases published in OC3 project
• Study of load cases where the wind and waves are misaligned

The project was done in collaboration with DHI and area of interest is chosen by DHI in connection with the modeling work that DHI is carrying out in North Sea region where most of the offshore wind turbine work is concentrated. The wave modeling task is carried out under supervision of DHI and the existing wave model which covers the region of interest is provided by them for validation and sensitivity analyses before obtaining wave forces from the wave model outputs for use in aeroelastic code.

Outline of report

Chapter one handles the wave modeling task carried out in collaboration with DHI, which can also be considered as the assessment of wave conditions for the offshore wind turbine. Chapter two explains the development of the aeroelastic model for the offshore turbine which includes implementations of models for turbine structure, aerodynamic loads, wave loads and control system. Chapter three presents the comparison of the model with test cases published in OC3 report serving as the code verification step for aeroelastic code development. After verification cases, in Chapter four the code is run for some defined cases for analysis of loads in FINO1 location as well as analysis of wind-wave directionality. Finally, the conclusions of the project is presented in Chapter five.
Chapter 1

Numerical Wave Modeling

The purpose of the wave modeling task in this project is to obtain the wave input for the simple aeroelastic model in order to be able to simulate the combined wind and wave loading on the turbine as realistic as possible in the area of focus, i.e. the German Bight area in the North Sea. This focus area is one of the important areas in the North Sea region that is affected by rough sea states since it covers a number of completed, ongoing and planned wind farm constructions as well as research platforms. Therefore, it becomes very important that appropriate wave forecasts are available for this area in order to predict the extreme sea states accurately with regard to safety of wind turbines and with regard to the assumptions that has to be made for their design. This modeling study is important since it also helps having a detailed understanding of the wave fields and the physical phenomena underlying these wave fields during the extreme sea states in North Sea. This will provide the essential background to be able to analyze the contribution of wave loads and their directions on the results obtained from the aeroelastic model of the offshore wind turbine.

MIKE 21 SW, which is the software used in the wave modeling task of this project, is a wave energy spectral wave model or a wave action spectral model that is the most commonly used type of wave model employed in modeling large-scale wave motion. This type of model assumes that a random sea-state is composed of infinite number of linear waves where wave height is a function of wave frequency and the direction of propagation of the wave [7].

In this chapter, first, a theoretical background on wind-driven waves and spectral wave model is given explaining the wave spectra and wave action conservation equation. Next, a brief presentation of the MIKE 21 SW software is given. Finally, the wave model set up by MIKE 21 SW is described and the sensitivity analysis of this model is presented.

1.1 Theoretical background on Wind-generated waves

1.1.1 Basic Concepts

Wind waves are generated by the action of the wind on the sea surface. For deep water, wind waves at the sea surface depend on three main factors: wind speed, duration and
fetch. Among these factors, duration refers to the period of time during which wind has blown over a given area and fetch refers to the distance of open water that the wind has blown over. By definition, wind-waves are characterized by the following:

- **Wave height** is the difference between the elevations of a crest and a neighboring trough
- **Wavelength** is the distance between two adjacent crests
- **Wave period** is the time interval between arrival of two consecutive crests
- **Wave propagation direction** is the direction in which waves travel

As wind blows over the sea surface three different types of wind waves develop over time

- Capillary waves or ripples
- Seas
- Swells

The formation of each type can be explained briefly as follows. When wind starts blowing over an initially calm sea surface, it first forms very small waves which are named as capillary waves where the restoring force is mainly surface tension and gravity is not very important. As wind raises further, waves get higher and at some point gravity replaces surface tension as the major restoring force forming waves called as seas. Seas are larger scale waves in the form of surface gravity waves. When the fetch and duration is sufficiently large, a sea state known as fully developed sea is reached. On the other hand, when waves are still growing under the effect of local wind field are referred to as wind sea. Wind sea generally travels parallel to the wind direction. When the wind direction and/or speed changes, a new wind sea system develops. The old waves are decoupled from the wind forcing, however they still remain existing. These waves are called swell. Swell consists of wind waves that are not or hardly effected by the local wind at that time and can travel over very long distances with a very little damping. Therefore, swells coming from other systems should be taken into account when wave model results are observed at a location \[8\].

### 1.1.2 Wave Spectra

Wind-generated waves in the real sea are generally three-dimensional. Therefore, a common approach is to represent the sea state by three-dimensional waves which are composed of infinite number of regular two-dimensional waves with different frequencies and directions. In this case, the wave spectrum for these waves become a function of both frequency \(f\) and wave direction \(\theta\).

The wave energy in this generalized case is
1.1. THEORETICAL BACKGROUND ON WIND-GENERATED WAVES

\[ \sigma^2 = \int_0^\infty \int_{-\pi}^\pi S_\eta(f, \theta) \, d\theta \, df \]  

(1.1)

**Figure 1.1:** Example of a wave spectra

where \( S_\eta(f, \theta) \) is the three-dimensional spectrum function. The quantity \( S_\eta(f, \theta) \, d\theta \, df \) represents the contribution of the waves with frequency and direction in the small rectangle \( d\theta df \) to the total wave wave energy which can be observed in Figure 1.1 [9].

1.1.3 Wave Action Density Conservation Equation

In order to describe a space-time varying sea state, the basic equation is a balance equation for wave action density which is also the governing equation of MIKE 21 SW [8]. The equations presented here are taken from [10].

Wave action density \( N \) is proportional to the wave energy density \( E \) and is defined as follows

\[ N(\sigma, \theta) = \frac{E(\sigma, \theta)}{\sigma} \]  

(1.2)

The independent parameters in \( N \) are the relative intrinsic frequency \( \sigma = 2\pi f \) and the direction of wave propagation \( \theta \). The relation between relative angular frequency \( \sigma \) and the absolute angular frequency \( \omega \) is given by the linear dispersion relation

\[ \sigma = \sqrt{gktanh(kd)} = \omega - k \cdot U \]  

(1.3)
where \( g \) is gravity, \( d \) is water depth, \( U \) is the current velocity vector and \( k \) is the wave number vector which has \( k \) as magnitude and \( \theta \) as direction. The fundamental wave action density balance equation reads

\[
\frac{\partial N}{\partial t} + \nabla \cdot (\nu N) = \frac{S}{\sigma}
\] (1.4)

where \( t \) is time, \( N(x, \sigma, \theta, t) \) is the wave action density, \( x(x, y) \) is the Cartesian coordinates, \( \nu \) is the group velocity in the four-dimensional phase space \( x, \sigma \) and \( \theta \), and \( \nabla \) is the four-dimensional differential operator in the \( x, \sigma, \theta \) space.

On the right hand side of equation (1.4), \( S \) is the energy source term consists of superposition of the source functions of various physical phenomena

\[
S = S_{\text{in}} + S_{\text{nl}} + S_{\text{ds}}
\] (1.5)

The first source term \( S_{\text{in}} \) denotes the energy transfer from the wind into the wave field. Wind input represents the only source of energy for generation of sea surface waves for the time scales considered in the balance equation. The transfer of energy from the wind to the wave field is approximately proportional to the square of wind speed which makes the quality of wind field input used in the numerical wave model very important for the quality of the wave simulations [8]. The second source term \( S_{\text{nl}} \) represents weakly nonlinear wave-wave interactions. The theory of non-linear wave-wave interactions is not inside the scope of this project, but their effects need to be mentioned here. The most important effect of these interactions is that they shift energy from higher to lower frequencies which results in waves dominating the sea becoming longer and longer as time passes from the onset of the wind field. The third term \( S_{\text{ds}} \) denotes the dissipation of wave energy from the spectrum. The dissipation process can occur by three different processes [8].

- **Whitecapping** is the main dissipation process in open seas and under deep-water conditions. It is the name given for the breaking of wave after they grow up to a certain height. Therefore, this process limits the wave growth in open seas.
- **Surf breaking** is relevant only for very shallow water and therefore is not of importance for this study
- **Wave-bottom interaction** is the dissipation of wave energy by the interaction of waves with the bottom.

### 1.1.4 Integral Wave Parameters

As mentioned earlier, spectral wave model describes the sea states in the form of evolution of the wave spectrum. For a detailed structural analysis of the wind turbine, use of wave spectra itself is of high importance. In the wave model analysis presented in this chapter the interest is in some parameters that can be derived from the spectrum and
which are mostly provided as additional model outputs. The four main parameters which are most commonly used to describe sea states will be given in this section. These four parameters which are also used in this study are

- significant wave height \( H_{m0} \)
- peak wave period \( T_p \)
- mean wave period \( T_{02} \)
- mean wave direction \( \theta_m \)

The following expressions for the parameters are taken from [8]. The first three of these parameters are expressed using the spectral moments of the spectrum distribution. The n-th moment of the spectrum is defined as

\[
m_n = \int_0^\infty \int_0^\infty f^n S_\eta(f, \theta) df d\theta
\]  

(1.6)

The significant wave height is historically defined as the average of the largest third of the wave height recordings and referred to as \( H_{1/3} \). In spectral analysis, spectral wave height is defined by the zero order moment \( m_0 \) as

\[
H_{m0} = 4\sqrt{m_0}
\]  

(1.7)

Second wave parameter, peak period, is the period corresponding to the peak frequency of the spectrum. The mean wave period \( T_{02} \) on the other hand represents the mean period between two zero upcrossings and given by

\[
T_{02} = \sqrt{\frac{m_0}{m_2}}
\]  

(1.8)

1.2 MIKE21 SW: Spectral modeling software

MIKE 21 SW is a state-of-the-art third generation spectral wind-wave model developed by DHI Water and Environment that is used for assessment of wave climates in offshore and coastal areas. It simulates the growth, decay and transformation of wind-generated waves and swell in offshore and coastal areas. It is widely used in the areas of design of offshore, coastal and portal structures where accurate prediction of wave loads are of the highest priority for safety and economical considerations. For historical storms, where measured data is often not available for long periods to allow estimation of the extreme sea states accurately, MIKE 21 SW can be supplemented with measured hindcast data to simulate the wave conditions. A short description of the software can be seen in Appendix D.

MIKE 21 SW has two different formulations:

- Fully spectral formulation
• Directional decoupled parametric formulation

The model takes into account the following phenomena

• Wave growth by the action of wind
• Non-linear wave-wave interaction
• Dissipation due to white capping
• Dissipation due to bottom friction
• Dissipation due to depth-induced wave breaking
• Refraction and shoaling due to depth variations
• Wave-current interactions
• Effect of time-varying water depth and flooding and drying
• Effect of ice coverage on the wave field

1.2.1 Model Inputs

The model inputs are classified into these following categories

• Domain and time parameters
  – computational mesh
  – coordinate type(Cartesian or spherical)
  – simulation length
  – overall time step

• Equations
  – discretization and solution technique
  – formulation type
  – frequency and directional discretization
  – number of time steps

• Forcing parameters
  – water level data
  – current data
  – wind data
  – ice data
1.2. MIKE21 SW : SPECTRAL MODELING SOFTWARE

- Source function parameters
  - non-linear energy transfer
  - wave breaking (shallow water)
  - bottom friction
  - white capping

- Initial conditions
  - zero spectrum
  - empirical data
  - data file

- Boundary conditions
  - closed boundaries
  - open boundaries

Besides these input parameters mentioned above, it is also important to provide the model with an appropriate mesh in order to obtain reliable results. The important points in setting up the mesh is choosing the area to be modeled properly and setting the right resolution of the bathymetry.

Among all the inputs, using accurate hindcast and forecast wind data are of highest importance for large-scale wave models like the one used in this project, since wind is the main driving force in MIKE 21 SW.

Another very important point for the quality of model results is the resolution of frequency and direction for the solution of the wave spectra.

1.2.2 Model Outputs

In MIKE 21 SW for each time step and at each mesh point four types of output can be obtained. Among these, the output types of our interest are

- Integral Wave Parameters
- Directional-frequency wave spectra at selected grid points or areas, direction spectra and frequency spectra

The model outputs that were of importance in this project are

- significant wave height $H_{m0}$
- peak wave period $T_p$
- mean wave period $T_{02}$
- mean wave direction $\theta_m$
1.3 MIKE21 SW UKNS Wave Model

The initial wave model is an existing regional model provided by DHI which covers the United Kingdom and North Sea area (UKNS model). First, the outputs of the existing model is obtained, then a sensitivity analysis is carried on the model with respect to wind forcing input, boundary conditions and mesh resolution. Finally, all these different model results are compared against measured data for validation and comparison of accuracy purposes. The aim of the sensitivity analysis was to check the effect of changes in each parameter on model results and therefore to attempt to find ways of improving accuracy of the wave input to be used in the aeroelastic code.

Figure 1.2 shows the UKNS model domain and bathymetry. The area marked by red borders shows the German Bight area on which this project has focus. The boundaries of the model can be seen in Figure 1.3. The east boundary (E) is closed and the north (N), south (S) and west (W) boundaries are defined by wave action spectrum varying in time and along line that is obtained from the global North Atlantic model. The model is run using the fully spectral formulation for the period of May 2009 using a solution time step size of 120 seconds. The model outputs include the integral parameters as well as the full spectra at several locations including at FINO1 station and they are saved every 1 hour. The number of frequencies and directions of the spectral solution are 25 and 24 respectively.

In the following section the sensitivity analyses carried on UKNS model is presented and it is followed by the conclusion of the analyses.

Figure 1.2: UKNS model domain and German Bight area
1.3. MIKE21 SW UKNS WAVE MODEL

1.3.1 Sensitivity Analysis of model results

In order to obtain accurate wave data to be used in the load calculations of the turbine, accuracy/sensitivity of the existing UKNS model results were needed to be analyzed against some reliable observations in the given area which covers a sufficiently long period of time. The observations used in the sensitivity analyses are the measurements taken from FINO1 research platform which is located in the German Bight area, 45 km north of the island Borkum. The sensitivity/accuracy of the model is tested against the FINO1 measurements with respect to these three parameters

- **Wind forcing:** STORM vs. WATCH wind model data
- **Wave boundary conditions:** With boundary input vs. with closed boundaries
- **Wave model mesh resolution:** Regional low resolution model vs. local high resolution model

All three analyses are performed for the period May 2009 and for each analysis all other model inputs and parameters are kept the same except for the sensitivity parameter that is tested. The results of the analyses are presented and discussed in the following sections. The discussions are mainly based on statistical analysis performed on the comparisons between model results and FINO1 observations and effort has been given on relating these statistical analyses with physical explanations. The definitions of statistical parameters used in the statistical analyses are given in Appendix A. Among these parameters, the ones of most interest for the discussions are scatter index (SI), BIAS, correlation coefficient (CC) and peak ratio (PR). Before going into details of the analyses some brief information about FINO1 station and the data available from this station is given in the next section.

Figure 1.3: Boundary definitions of the UKNS model
1.3.1.1 FINO1 Observations

The research platform FINO1 was erected during summer 2003 in the North Sea, 45 km north of the island Borkum. Since then, the station performs and processes meteorological and oceanographic measurements and these data are available to project partners, research centers and also for commercial use to any organization who is interested for a fee.

In the analyses, wind and wave measurements of FINO1 data were used. Wind data consists of measurements of wind speed and direction. At the station, wind speed measurements are taken at heights between 33 m CD and 100 m CD in 10 m intervals. The wind data available for our use from the station is 10-minute averaged wind speed and wind direction data.

The wave measurements include all relevant sea state parameters such as significant wave height, period, direction, wave spectra etc. as well as maximum wave heights, periods and lengths. The measurements are performed using a wave rider buoy at a distance about 200 m from the platform and a Wave-Radar. The averaging period of data is 30 minutes and 3 minutes for wave rider buoy and for wave radar, respectively. Among the different sea state parameters, the ones used in the comparisons are significant wave height, mean wave period, peak wave period and mean wave direction.

1.3.1.2 Sensitivity with respect to wind forcing input

The sensitivity of the model against different wind forcing input is tested by running two identical setups which both have the same inputs and parameters except for the wind forcing input. The existing model runs for the period May 2009 and uses STORM wind model data as wind forcing input. In order to test the sensitivity of the model, this input is replaced by WATCH wind model data which covers the same time period as STORM data and the results obtained from both models are compared against FINO1 observations for that period. Analyzing sensitivity of the model with respect to wind input is important since wind input is the main driving force of the wave model. Therefore, before comparing the model results it is important to take a look at the differences in the two wind forcing inputs in order to be able to make a better reasoning of the differences in the wave parameters computed by each model.

Comparison of wind inputs

In order to be able to comment on the differences between the two wave model outputs for two different wind inputs, it is important to know how the two wind model data differ from each other first. For this reason, before comparing the wave model outputs, the two wind model inputs, namely WATCH and STORM data are compared with each other. The comparison is done by carrying out a statistical analysis for each wind model data which compares the 10-min averaged wind speeds ($U_{10}$) and wind directions ($U_{dir}$) of each model against FINO1 wind observations. In such a comparison, it is important to note that the measured data from FINO1 station cannot be directly used. This is due to the fact that just like many available numerical wind models, STORM and
1.3. MIKE21 SW UKNS WAVE MODEL

WATCH models specify wind speed at an elevation of 10 m above mean sea level whereas the measured wind speeds are taken at higher elevations [12]. Therefore, the wind speed measurements from FINO1 station need to be corrected to elevation of 10 m to be used in the comparisons. In this report, the measurements at the elevation of 33 m are used and corrected according to the following relation from [12] assuming moderate wave age.

\[
\frac{U_{10}}{U_z} = \left(\frac{10}{z}\right)^{0.108} \tag{1.9}
\]

where \(U_{10}\) is the corrected wind speed at the elevation of 10 m, \(U_z\) is the measured wind speed at the given height \(z\) and the value 0.108 is the power for moderate wave age.

<table>
<thead>
<tr>
<th>QIs</th>
<th>(U_{10})</th>
<th>(U_{dir})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>STORM</td>
<td>WATCH</td>
</tr>
<tr>
<td>MEAN</td>
<td>6.99</td>
<td>7.01</td>
</tr>
<tr>
<td>BIAS</td>
<td>-0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>RMSE</td>
<td>1.80</td>
<td>1.75</td>
</tr>
<tr>
<td>SI</td>
<td>0.25</td>
<td>0.24</td>
</tr>
<tr>
<td>CC</td>
<td>0.82</td>
<td>0.85</td>
</tr>
<tr>
<td>PR</td>
<td>0.71</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Table 1.1: Quality Indices for STORM and WATCH model wind inputs compared against FINO1 observations

Table 1.1 shows the quality indices obtained from the statistical analysis of the wind model data against the corrected observations. The time series comparisons for \(U_{10}\) and \(U_{dir}\) can be seen in Figure B.1. From Table 1.1, the scatter index (SI) for wind speeds show that they are in good agreement with the observations as their SI’s are within the acceptable range of 0.20-0.25. Comparing the peak ratios (PR) for wind speeds show that STORM model underestimates the wind speeds, whereas WATCH model overestimates. By definition of PR, a PR value below 1 indicates an underestimation while above 1 indicates overestimation of the model predictions compared against observations. As it will be seen in the following comparisons of wave parameters, this fact will be reflected on the significant wave heights as they are strongly affected by the wind speed input. For wind directions, comparing the SI values in Table 1.1 shows that both wind models are in good agreement with the wind direction observations having SI values around 0.15. The CC values show that the WATCH wind directions are in better correlation with the observations than STORM model, although the BIAS values for WATCH wind directions show a higher mean difference. Another comparison for wind directions can be made by rose plots presented in Figure 1.4. Comparing these rose plots it can be said that the dominating wind direction is better represented in STORM model. Overall, STORM wind directions and speeds are seen to be better as a wind input to wave model.
CHAPTER 1. NUMERICAL WAVE MODELING

Comparison of wave model outputs Table 1.2 shows the summary of the quality indices of the four main integral wave parameters obtained by each model compared against FINO1 observations. The comparisons can also be seen in time series in Figure B.2. For significant wave height, one observation that can be done from the time series and peak ratios is that the wave heights are predicted higher with WATCH wind input and the opposite holds when STORM wind input is used. This is expected since it was discussed previously that the wind speeds are higher for WATCH wind input and wind speeds have a significant influence on the predicted wave heights. Another important observation from wave height time series in Figure B.2 is that during the period between May 9 and 12 both models predict the wave heights higher than the observations. This difference can be explained by the existence of swell included through the model boundary conditions. For peak wave period there are also some observations worth mentioning. The time series comparison for peak period in Figure B.2 shows a jump during two intervals, one between May 2-4 and the other one between May 10-12. These can also explained with the existence of swell from the boundaries that was predicted by the larger North Atlantic model. From the statistical parameters and time series comparisons it is seen that the remaining two wave parameters, i.e. mean wave period and mean wave direction show a
good agreement with the FINO1 observations. As expected these two parameters do not show a significant amount of sensitivity with respect to wind input.

<table>
<thead>
<tr>
<th>Qls</th>
<th>$H_{m0}$</th>
<th>$T_p$</th>
<th>$T_{02}$</th>
<th>MWD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>STORM</td>
<td>WATCH</td>
<td>STORM</td>
<td>WATCH</td>
</tr>
<tr>
<td>MEAN</td>
<td>1.09</td>
<td>1.09</td>
<td>6.45</td>
<td>6.43</td>
</tr>
<tr>
<td>BIAS</td>
<td>0.09</td>
<td>0.34</td>
<td>0.48</td>
<td>0.61</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.26</td>
<td>0.43</td>
<td>2.81</td>
<td>2.73</td>
</tr>
<tr>
<td>SI</td>
<td>0.22</td>
<td>0.25</td>
<td>0.43</td>
<td>0.41</td>
</tr>
<tr>
<td>CC</td>
<td>0.91</td>
<td>0.93</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>PR</td>
<td>0.81</td>
<td>1.40</td>
<td>0.86</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Table 1.2: Quality Indices for model with STORM and WATCH wind inputs compared against FINO1 observations

Figure 1.5: Rose plots of $H_{m0}$ vs. MWD, sensitivity with respect to wind input

The comparison of significant wave heights and mean wave directions can also be seen in detail in Figure 1.5 in the form of rose plots.
1.3.1.3 Sensitivity with respect to wave boundary conditions

The second sensitivity of our interest is the sensitivity of the model results with respect to different wave boundary conditions. This sensitivity is tested by comparing two different cases of boundary conditions. First case is our reference wave model which uses wave action spectra varying in time and along line as boundary conditions. In the second case, these boundary conditions are replaced by closed boundary conditions while keeping all other inputs and parameters same as the reference model. The locations of the boundaries were shown on the model domain in Figure 1.3. The definitions of the boundary conditions for each case are shown in Table 1.3.

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Model with open boundaries (reference model)</th>
<th>Model with closed boundaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (East)</td>
<td>Closed</td>
<td>Closed</td>
</tr>
<tr>
<td>N (North)</td>
<td>Wave action spectrum, varying in time and along line</td>
<td>Closed</td>
</tr>
<tr>
<td>W (West)</td>
<td>Wave action spectrum, varying in time and along line</td>
<td>Closed</td>
</tr>
<tr>
<td>S (South)</td>
<td>Wave action spectrum, varying in time and along line</td>
<td>Closed</td>
</tr>
</tbody>
</table>

Table 1.3: Boundary condition definitions for open and closed boundary model

The main difference between open and closed boundary conditions is that open boundaries include information of swells coming from the boundaries whereas closed boundary setup only includes the waves generated by the given wind field over the domain. This fact will be observed in the following comparison section of the model results against FINO1 observations.

Comparison of wave model outputs  Both models cover the period of May 2009 and their results are compared against the FINO1 station observations for that same period. The quality indices obtained from the comparisons of the wave parameters for each model against observations are presented in Table 1.4. Comparisons of the wave parameters are also shown in time series in Figure B.3. Time series of significant wave heights shows that the high prediction of wave heights during May 9-12 is indeed due to the swell coming from outside of the domain since closed boundary setup is matching with the observations while open boundary gives slightly higher wave heights.
### Table 1.4: Quality Indices for model with open and closed boundaries compared against FINO1 observations

Comparing the correlation coefficients and time series for both peak and wave periods show that with open boundaries periods are modeled with better agreement to the observations. It is also seen from the time series that the jumps in the peak wave period mentioned in the previous sensitivity analysis (May 2-4 and 10-12) can be explained by the swell included in the boundary information from North Atlantic model. The effect of swell can also be observed from the wave spectra shown in Figure 1.6.

![Wave spectra showing swell and waves](image)

**Figure 1.6:** Wave spectra showing swell and waves

Figure 1.6(a) shows the two wave systems one coming from westerly direction and the other one from northwest direction during May 3rd where there is swell present (See the $H_{m0}$ and $T_p$ time series in Figure B.3). Figure 1.6(b) on the other hand represents the case of wind sea developed by the current wind conditions on May 6th where there is no effect of swell present (See the $H_{m0}$ and $T_p$ time series in Figure B.3). The model with open boundaries is able to capture the jumps in the peak period since it includes the swell whereas closed boundary model ignores these jumps. Overall, this sensitivity test demonstrates very well the importance of including boundary conditions to the wave model.
CHAPTER 1. NUMERICAL WAVE MODELING

model. At this point, it is important to mention that accurate predictions of wave heights and periods with a model with open boundaries depend also on the quality of the model from which the boundary information is extracted.

Figure 1.7: Rose plots of $H_m0$ vs. MWD, sensitivity with respect to boundary conditions

1.3.1.4 Sensitivity with respect to wave model resolution

In order to test the sensitivity/accuracy of the model against change in model mesh resolution, a local high resolution model is made for the German Bight Area and then the results are compared with the ones from the existing regional (UKNS) model. The domain for the local high resolution model is shown in Figure 1.8. It mainly covers German Bight area as well as small part of Denmark.

The location of the local high resolution model domain was shown on the existing regional (UKNS) model domain previously in Figure 1.2. The main purpose of this sensitivity/accuracy test is to develop a higher resolution model for the German Bight area and investigate how the accuracy of wave condition predictions can be improved by increasing the resolution in our area of interest. Creating such a model is indeed very important since the domain covers an important area for offshore wind turbine
installations where accurate predictions of wave conditions are necessary to predict the extreme loads on the turbine structures.

Figure 1.9 shows the element mesh and the bathymetries of the local model. It can be observed that near the shoreline the mesh is fine and towards the outside region two layers with coarser resolutions are defined in order to maintain a gradual transition from fine to coarse resolution. In the shoreline layer, the boundary is extended a little towards the north in order to include our points of interest which are enclosed inside a much finer small area. The area enclosing the points of interest is made around 2 times finer than the shoreline layer since more accurate results are needed for comparisons at those points where observed data have been collected.

The aim of the overall local model mesh is to have a resolution higher than UKNS model. The order of the increase in the resolution of local model with respect to the UKNS model is compared in terms of the length parameters of the models in Table 1.5.

<table>
<thead>
<tr>
<th>Region</th>
<th>Model length parameter [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Local</td>
</tr>
<tr>
<td>Shoreline region</td>
<td>1500-2000</td>
</tr>
<tr>
<td>First transition</td>
<td>3000-4000</td>
</tr>
<tr>
<td>Second transition</td>
<td>4000-5000</td>
</tr>
<tr>
<td>Outer region</td>
<td>5000-7000</td>
</tr>
</tbody>
</table>

Table 1.5: Comparison of mesh lengths, regional vs. local
Once the mesh is made the local high resolution model setup is prepared such that all the model parameters and inputs except the mesh are kept the same as the existing UKNS wave model setup for the May 2009 simulation which is using STORM wind data. Both regional and local model wave parameters are compared against the FINO1 observations for the same period of May 2009. The quality indices for the wave parameters obtained from both models are presented in Table 1.6. The wave parameters are also presented in time series in Figure B.4. Both quality indices and time series show that the model outputs do not show a significant sensitivity to increased mesh resolution. The significant wave heights and mean wave directions can also be observed in the form of rose plot in Figure 1.10.

<table>
<thead>
<tr>
<th>QIs</th>
<th>$H_{m0}$ Regional</th>
<th>$T_p$ Regional</th>
<th>$T_{o2}$ Regional</th>
<th>MWD Regional</th>
<th>$H_{m0}$ Local</th>
<th>$T_p$ Local</th>
<th>$T_{o2}$ Local</th>
<th>MWD Local</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEAN</td>
<td>1.09</td>
<td>6.45</td>
<td>4.43</td>
<td>241.66</td>
<td>1.09</td>
<td>6.45</td>
<td>4.43</td>
<td>241.66</td>
</tr>
<tr>
<td>BIAS</td>
<td>0.09</td>
<td>0.48</td>
<td>-0.35</td>
<td>-2.27</td>
<td>0.05</td>
<td>0.33</td>
<td>-0.44</td>
<td>-4.63</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.26</td>
<td>2.81</td>
<td>0.86</td>
<td>33.16</td>
<td>0.23</td>
<td>2.65</td>
<td>0.92</td>
<td>33.69</td>
</tr>
<tr>
<td>SI</td>
<td>0.22</td>
<td>0.43</td>
<td>0.18</td>
<td>0.14</td>
<td>0.21</td>
<td>0.41</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>CC</td>
<td>0.91</td>
<td>0.60</td>
<td>0.64</td>
<td>0.68</td>
<td>0.92</td>
<td>0.63</td>
<td>0.62</td>
<td>0.64</td>
</tr>
<tr>
<td>PR</td>
<td>0.81</td>
<td>0.86</td>
<td>0.66</td>
<td>1.00</td>
<td>0.79</td>
<td>0.89</td>
<td>0.60</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 1.6: Quality Indices for Local and Regional models compared against FINO1 observations
1.3. MIKE21 SW UKNS WAVE MODEL

Figure 1.10: Rose plots of $H_m0$ vs. $MWD$, sensitivity with respect to mesh resolution

1.3.2 Conclusion

The three sensitivity analyses show that the model results are mainly sensitive to wind forcing input. The results also show a considerable amount of sensitivity to boundary condition definitions. On the other hand, it is observed that the local high resolution did not make a significant improvement regarding the four main parameters of interest in the analyses. Besides the sensitivity comparisons, the comparison of model results against FINO1 observations provided also a way of validating the UKNS model at the given FINO1 location. As a result, the overall analysis provided essential information about the sea-states and about the various phenomena affecting these states during extreme events which will be useful in the load analysis of an offshore wind turbine under wave loading in German Bight area. At this point, it is important to note that this analysis covers a period of only one month which can be considered as a very selective data. Therefore, the conclusions drawn from these analyses should be used with care. It should be noted that for more appropriate statistical interpretations the period should be at least a couple of years. Nevertheless, based on this analysis a wave spectra at point FINO1 covering the period of May 2009 is obtained from the UKNS model which uses STORM data as wind forcing and open boundaries. This spectra will be used to generate the wave speed,
wave direction and water elevation time series which will then be used for calculations of wave force time series that will be used as wave force input to the aeroelastic code. The transformation from spectrum to time series and calculation of wave forces from these time series will be explained in the aeroelastic modeling section.
Chapter 2
Aero-elastic modeling

After the assessment of wave conditions around the offshore wind turbine, the second task is the development of an aeroelastic code which will combine the wave input obtained from the wave model with the wind input to simulate the dynamic response of the turbine under combined wind and wave loads. In order to be able to determine the dynamic wind and wave loads acting on the components of the wind turbine, an aeroelastic model for the turbine is needed. Aeroelastic model solves the so-called aeroelastic problem which comprises of solving the highly coupled structural and aerodynamic models of the turbine simultaneously. The reason why the two models are coupled is that the velocity of vibrating turbine components affect the relative velocity seen locally by the blade. This means that the loads calculated by the aerodynamic model depend on the deflections and velocities of the structure calculated by the structural model which again depend on the loads. This section presents development of a simple aeroelastic model for an offshore wind turbine which is to be used in the determination of combined wind and wave loads acting on an offshore wind turbine, which makes up the main analysis of this project. The offshore wind turbine that is used in the model is the "5-MW Reference Wind Turbine for Offshore System Development" which is also known as the "NREL offshore 5-MW baseline wind turbine". Using the specifications given for this turbine, the dynamic response of the turbine is modeled assuming 4 degrees of freedom, which are under the effect of combined aerodynamic (wind) and hydrodynamic (wave) loads.

The steps for setting up the aeroelastic model of the offshore wind turbine are as follows

- The necessary degrees of freedom are defined considering the purpose of the load analysis. In this project, the focus is on the loads on the support structure. Modal shape functions are used to define the degrees of freedom associated with the support structure, which reduces the total degrees of freedom needed to define the tower motion, therefore reducing the computational effort per time step.

- The resulting equation of motion for the chosen degrees of freedom is written based on the mass, stiffness and damping matrices obtained through principle of virtual work.
In order to solve the resulting non-linear equation of motion Runge-Kutta Integration method, which is a full non-linear time domain approach, is implemented.

To calculate the time-dependent aerodynamic loads that appear as external loads in the equation of motion, an unsteady BEM model is implemented as an aerodynamic model for the turbine.

The time-dependent wave loads in the equation of motion are determined using the wave time series obtained from the wave model in Chapter 1. To determine these wave forces acting on the support structure over the entire water depth, a simple discretized model for the tower and monopile structure is also set up. Using this model, wave loads at each water level are calculated through Morison’s equation from the time series of wave velocity and water elevation.

In the following subsections each of these steps above are explained in detail.

### 2.1 Turbine Specifications

All the turbine specifications relevant to this study are taken from the NREL report [13] that presents the complete definition of the 5-MW reference wind turbine. It is prepared by the U.S. Department’s National Renewable Energy Laboratory (NREL) and the study aimed at establishing realistic and standardized input data to be used in the assessments of both land and sea-based multi-megawatt turbines. Using these standard specifications for the turbine in the simple aeroelastic code makes it possible to compare the results of the model with the state-of-the-art aeroelastic offshore wind codes in the OC3 collaboration presented in [5].

The NREL turbine is a conventional three-bladed, upwind, variable-speed variable-pitch controlled turbine having the main properties that are presented in Table 2.1.

<table>
<thead>
<tr>
<th>Rating</th>
<th>5 MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor orientation, configuration</td>
<td>Upwind, 3 blades</td>
</tr>
<tr>
<td>Control</td>
<td>Variable speed, collective pitch</td>
</tr>
<tr>
<td>Drivetrain</td>
<td>High-speed, Multi-Stage gearbox (Ratio:97)</td>
</tr>
<tr>
<td>Rotor, Hub diameter</td>
<td>126 m, 3 m</td>
</tr>
<tr>
<td>Hub Height</td>
<td>90 m</td>
</tr>
<tr>
<td>Cut-in, Rated, Cut-out wind speed</td>
<td>3, 11.4, 25 m/s</td>
</tr>
<tr>
<td>Cut-in, Rated rotor speed</td>
<td>6.9, 12.1 rpm</td>
</tr>
<tr>
<td>Rated tip speed</td>
<td>80 m/s</td>
</tr>
<tr>
<td>Rotor mass</td>
<td>110000 kg</td>
</tr>
<tr>
<td>Nacelle mass</td>
<td>240000 kg</td>
</tr>
<tr>
<td>Blade mass (integrated, per blade)</td>
<td>17740 kg</td>
</tr>
<tr>
<td>Tower + monopile mass (integrated)</td>
<td>522620 kg</td>
</tr>
</tbody>
</table>

**Table 2.1:** Gross properties chosen for the NREL 5-MW baseline wind turbine
2.2. DYNAMIC STRUCTURAL MODEL OF THE TURBINE

The detailed specifications for the blade, nacelle, hub, tower, drivetrain and control system properties are taken from [13].

2.2 Dynamic Structural Model of the Turbine

The main purpose of having a dynamic structural model of a wind turbine is to be able to calculate the deflections and velocities of the turbine components in the time domain. Coupling the model with the aerodynamic model (unsteady BEM) which calculates the time dependent wind loads, the dynamic structural response and the loads of the entire construction can be calculated. For an offshore wind turbine, time dependent wave loads must also be included in the model.

2.2.1 Degrees of Freedom and Equation of Motion

In order to construct the dynamic structural model of the turbine, first thing to do is to determine the sufficient number of degrees of freedom that describe deformations realistically and to write down the equation of motion for those DOFs by setting up the correct mass, damping, stiffness matrices and the external force vector. The general form of the equation of motion for a discretized mechanical system can be written as

\[ M \ddot{x} + D \dot{x} + Kx = F_g \]  

(2.1)

where \( M \) is the mass matrix, \( D \) is the damping matrix, \( K \) is the stiffness matrix and \( F_g \) denotes the generalized force vector associated with the external loads.

The number of elements in vector \( x \) represents the number of DOFs of the system. In general, the number of degrees of freedom in a turbine model can vary from single DOF up to infinitely many number of DOFs theoretically, depending on the purpose and the required complexity of the model. In this project, the aim is to keep the model as simple as possible, focusing on the degrees of freedom on which the loads are to be analyzed. As mentioned before, the focus of the analysis is to observe the effect of combined wind and wave loads on the support structure of the turbine. Therefore, the main degrees of freedom will be associated with the tower and monopile structure of the turbine. Based on this, two main motions are chosen to model the tower deflections assuming stiff structure in the length direction and in torsion. One motion is associated with the side-to-side and the other one is associated with the fore-aft bending deflections of the tower. In order to be able to define these deflections realistically, the tower structure has to be discretized into many segments. This would result in a large number of DOFs for each tower motion, therefore the computational time needed at each time step to solve the equation of motion would increase.
2.2.1.1 Generalized coordinates

In order to avoid long computational time at each time step, use of modal shape functions method is employed when modeling the tower structure. Use of modal shape functions reduce the computational time at each time step by decreasing the number of degrees of freedom and therefore reducing the size of matrices to be solved in the equation of motion at each time step. In this method, the deflection of the tower and monopile structure is described by a limited number of parameters called generalized coordinates which define the amount of deflection through a shape function rather than having discretized points along the tower length as degrees of freedom. A deflection shape function is defined as linear combination of a few physically realistic functions that describes the deflection of the structure. The chosen deflection shapes often correspond to the eigenmodes of the structure with the lowest eigenfrequencies, which is also the case in this model. The generalized coordinates for both side-to-side and fore-aft motion of the tower are obtained using the first eigenmode shape of the modeled tower. To describe the side-to-side deflection of the tower, it is sufficient to use a single generalized coordinate, $G_Y$, defined by the first eigenmode shape of the tower as the associated degree of freedom which is excited by the distributed wave loads in $y$ direction only. The fore-aft motion on the other hand required two DOFs to describe the deflection in the $z$ direction since it is subject to the thrust force which acts as a point force applied from the tower top in addition to the distributed loads of waves in $z$ direction. As a result, the degrees of freedom, $G_{Z1}$ and $G_{Z2}$, are defined for deflection of tower due to wave loading in $z$ direction and point thrust force at tower top respectively, giving the total deflection of the tower in $z$ direction when summed up together since the system is linear. In addition to these three generalized coordinates describing the tower motion, the final degree of freedom is chosen as the rotation of the rigid rotor, adding up to 4 degrees of freedom in total for the structural model.

The resulting equation of motion becomes

$$
\begin{bmatrix}
\dddot{G}_Y \\
\dddot{G}_{Z_1} \\
\dddot{G}_{Z_2} \\
\ddot{\theta}
\end{bmatrix} +
\begin{bmatrix}
G_Y \\
G_{Z_1} \\
G_{Z_2} \\
\theta
\end{bmatrix}
\begin{bmatrix}
\dddot{G}_Y \\
\dddot{G}_{Z_1} \\
\dddot{G}_{Z_2} \\
\ddot{\theta}
\end{bmatrix} +
\begin{bmatrix}
G_Y \\
G_{Z_1} \\
G_{Z_2} \\
\theta
\end{bmatrix}
\begin{bmatrix}
F_{g,y} \\
F_{g,z_1} \\
F_{g,z_2} \\
F_{g,\theta}
\end{bmatrix} =
\begin{bmatrix}
F_{g,y} \\
F_{g,z_1} \\
F_{g,z_2} \\
F_{g,\theta}
\end{bmatrix}
$$

(2.2)

The mass, stiffness, damping matrices and the generalized forces in Eq. 2.2 are

$$
M =
\begin{bmatrix}
GM & 0 & 0 & 0 \\
0 & GM & 0 & 0 \\
0 & 0 & GM & 0 \\
0 & 0 & 0 & I_R + I_G
\end{bmatrix}
$$

(2.3)
2.2. Dynamic Structural Model of the Turbine

\[ K = \begin{bmatrix} GK & 0 & 0 & 0 \\ 0 & GK & 0 & 0 \\ 0 & 0 & GK & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  
\[ D = \begin{bmatrix} GD & 0 & 0 & 0 \\ 0 & GD & 0 & 0 \\ 0 & 0 & GD & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  
\[ F_g = \begin{bmatrix} F_{g,\text{wave}_y} \\ F_{g,\text{wave}_z} \\ T \\ M_r - (N_{\text{gear}}M_g) \end{bmatrix} \]

where the generalized mass (GM), stiffness (GK), damping (GD) and force \( F_{g,\text{wave}} \) are obtained for the chosen first eigenmode shape through the relations:

\[ GM = \int_0^H u(x)m(x)u(x) \, dx \]  
\[ GK = \omega_1^2 GM \]  
\[ GD = (GM)2\zeta\omega_1 \]  
\[ F_{g,\text{wave}_y} = \int_0^H p_{\text{wave}_y}(x)u(x) \, dx \]  
\[ F_{g,\text{wave}_z} = \int_0^H p_{\text{wave}_z}(x)u(x) \, dx \]

In equation 2.6, \( T \) is the thrust force at the tower top, \( M_r \) is the rotor torque, \( M_g \) is the generator torque on the high speed shaft and \( N_{\text{gear}} \) is the gearbox ratio.

### 2.2.1.2 Calculation and use of eigenmodes

As introduced earlier, the physical deflections of the tower is related to the generalized coordinates through shape functions. The relation can be written as

\[ u(x) = \sum_{j=1}^N GX_j u_j(x) \]

where the factors \( GX_j \) are the generalized coordinates denoting the contribution of the shape function \( u_j \) to the total deflection.

An efficient choice for shape functions is using eigenmode shape functions which are described by their own distinct eigenfrequencies. Using the eigenmodes corresponding to the first 3-4 lowest frequencies is sufficient to model the tower response up to a high degree.
of accuracy, since it covers the most important low and middle frequency response and omits the small amplitude high frequency response. In this model, the same eigenmode shape which includes only the first eigenfrequency is used for the generalized coordinates \((GY GZ_1 \text{ and } GZ_2)\) since the tower is axisymmetric and therefore deflects the same way in both side-to-side and fore-aft directions. In order to calculate the eigenmodes, a method for calculating the deflections of a beam under bending has to be employed. The method used in this project is the numerical algorithm described in [14]. This algorithm assumes the tower as a cantilever beam discretized into \(N\) points and calculates the bending moments and deflections using slope angles and curvature formulas.

Having the algorithm for deflections of the beam, the method given in [15] for calculating the eigenmodes is implemented. All the equations presented here are taken from [15]. The relation between the load and deflection can be written in matrix form as

\[
\mathbf{u} = \mathbf{FP} \tag{2.13}
\]

where \(\mathbf{F}\) is the flexibility matrix of size \(NxN\), \(\mathbf{u}\) is the deflection vector of size \(Nx1\) and \(\mathbf{P}\) is the load vector of size \(Nx1\). Here it should be noted that the first point is clamped therefore \(u_1 = 0\). The columns of flexibility matrix are the deflection vectors for a unit loading at the corresponding point while all other loads are zero. From the definition of an eigenmode, at the eigenfrequency \(\omega\) the corresponding eigenmode is in static equilibrium with the inertia forces which is given by the following expression

\[
\mathbf{P} = \omega^2 \mathbf{M} \mathbf{u}_{ev} \tag{2.14}
\]

where \(\mathbf{u}_{ev}\) is the modeshape vector at the eigenfrequency \(\omega\) and \(\mathbf{M}\) is the mass matrix with the discretized masses of the tower segments in the diagonal.

Thus, combining (2.13) and (2.14) the equilibrium can be written as

\[
\mathbf{u}_{ev} = \omega^2 \mathbf{F} \mathbf{M} \mathbf{u}_{ev} \tag{2.15}
\]

Then, the standard eigenvalue problem is obtained as

\[
[\mathbf{FM}] \mathbf{u}_{ev} = \lambda \mathbf{u}_{ev} \tag{2.16}
\]

where \([\mathbf{FM}]\) is \(\mathbf{F} \cdot \mathbf{M}\) and \(\lambda = 1/\omega^2\)

The solution to this eigenvalue problem given in eq. (2.16) gives the first \(N\) eigenfrequencies and their corresponding mode shapes. In this aeroelastic model, only the first eigenmode shape function corresponding to the first eigenfrequency is used as the deflection shape for each of the generalized coordinates. Implementing the above described methods, the first eigenfrequency of the tower structure including the top mass for nacelle and rotor weight is obtained as 0.29 Hz, which compares well with the first eigenfrequency computed in OC3 report [5] as 0.28 Hz. The first eigenmode shape that is used as the deflection shape to define the tower degrees of freedom can be seen in Figure 2.1.
2.3. CALCULATION OF WIND LOADS

Aeroelastic loads acting on the turbine blades are computed by an unsteady BEM model. This model takes into account the unsteadiness of the wind seen by the rotor due to the following:

- Atmospheric turbulence
- Wind Shear

Figure 2.1: First eigenmode shape of the tower structure including top mass, at first eigenfrequency = 0.29 Hz

2.2.2 Dynamic response of the turbine: Runge-Kutta Integration Scheme

The dynamic response of the turbine is calculated using a Runge-Kutta-Nysőrm integration scheme.

Using the general form of the equation of motion given in equation \ref{eq:2.1} the second time derivative \( \ddot{x} \) can be obtained as a function of \( t, x \) and \( \dot{x} \):

\[
\ddot{x} = M^{-1}(F - D\dot{x} - Kx) = g(t, x, \dot{x}) \quad (2.17)
\]

The algorithm requires computing the values for intermediate time steps, therefore it is important to have a \( g \) function implemented, which can run for each of these time steps. The \( g \) function includes calling the unsteady BEM function each time \( g \) function is called in order to calculate the contribution of external loads to the \( g \) function.

2.3 Calculation of Wind Loads

Aeroelastic loads acting on the turbine blades are computed by an unsteady BEM model. This model takes into account the unsteadiness of the wind seen by the rotor due to the following:

- Atmospheric turbulence
- Wind Shear
A = \frac{\Delta t}{2} \ddot{x}^n

b = \frac{\Delta t}{2} (\ddot{x}^n + \frac{1}{2}A)

B = \frac{\Delta t}{2} g(t^{n+1}, x^n + b, \dot{x}^n + A)

C = \frac{\Delta t}{2} g(t^{n+1}, x^n + b, \dot{x}^n + B)

d = \Delta t (\ddot{x}^n + C)

D = \frac{\Delta t}{2} g(t^{n+1}, x^n + d, \dot{x}^n + 2C)

and the final update:

t^{n+1} = t^n + \Delta t

x^{n+1} = x^n + \Delta t(\ddot{x}^n + \frac{1}{2}(A + B + C))

\dot{x}^{n+1} = \dot{x}^n + \frac{1}{2}(A + 2B + 2C + D)

\ddot{x}^{n+1} = g(t^{n+1}, x^{n+1}, \dot{x}^{n+1})

Figure 2.2: Runge-Kutta-Nystörm Integration Scheme Algorithm

- Presence of tower
- Changes in tilt, yaw, cone and pitch angles of the turbine over time
- Velocity of the vibrations of tower over time

2.3.1 Relative velocity and velocity triangle

The relative velocity seen by a point on a blade is shown on a velocity triangle in Figure 2.3.

The vector representation of relative velocity can be written as

\begin{align*}
V_{rel} &= V_0 + V_{rot} + W \\
\Rightarrow \begin{pmatrix} V_{rel,y} \\ V_{rel,z} \end{pmatrix} &= \begin{pmatrix} V_y \\ V_z \end{pmatrix} + \begin{pmatrix} -\omega \cos \theta_{cone} \\ 0 \end{pmatrix} + \begin{pmatrix} W_y \\ W_z \end{pmatrix} \\
&= \begin{pmatrix} V_y - \omega \cos \theta_{cone} + W_y \\ V_z + W_z \end{pmatrix}
\end{align*}

Here, $V_0$ is the undisturbed wind velocity seen by the blade, $V_{rot}$ is the rotational velocity at the corresponding radial position of the point on the blade and $W$ is the induced velocity vector.
2.3. CALCULATION OF WIND LOADS

From the velocity triangle the angle of attack $\alpha$ is defined as

$$\alpha = \phi - (\beta + \theta_p)$$  \hfill (2.19)

where

$$\tan \phi = \frac{V_{rel,z}}{V_{rel,y}}$$  \hfill (2.20)

In order to compute the angle of attack $\alpha$ locally on along the blade, induced velocity $W$ has to be computed. The essence of unsteady BEM lies within the calculation of induced velocities to determine the local angles of attack. Unsteady BEM takes into account the deflection in the wake due to the induced velocity normal to the rotor plane which is caused by the thrust force generated by the pressure drop across the rotor. The equations that are used in the unsteady BEM model of this project for normal and tangential induced velocities can be derived from Glauert’s relation between thrust and induced velocity as

$$W_n = W_z = \frac{-BL \cos \phi}{4\pi \rho r F |V_0 + f_g n(n \cdot W)|}$$  \hfill (2.21)

$$W_t = W_y = \frac{-BL \sin \phi}{4\pi \rho r F |V_0 + f_g n(n \cdot W)|}$$  \hfill (2.22)

2.3.2 Coordinate System Transformation

In equation $V_0$ is the incoming wind velocity relative to a point on a blade. However, the incoming wind velocities are normally defined relative to the bottom of the turbine tower, e.g. relative to the ground or mean sea level for onshore and offshore turbines respectively. Therefore, the undisturbed wind velocities have to be transformed from the fixed coordinate system at the bottom of the tower to the coordinate system attached to the blades. To make this transformation, the position of a point at any section along the blade also has to be known relative to a fixed coordinate system.

Figure 2.4 shows the model described by four coordinate systems. System 1 is the fixed coordinate system that is placed at mean sea level. System 2 is placed at the nacelle and is able to rotate about x-axis with the angle $\theta_{yaw}$ and about the y-axis with the angle $\theta_{tilt}$. System 3 is placed to the rotating shaft which rotates about z-axis with angle $\theta_{wing}$. Finally, system 4 is aligned to one of the blades that can be rotated about y-axis with angle $\theta_{cone}$.
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Figure 2.4: Four coordinate systems describing the wind turbine, \([14]\)

A vector in coordinate system \(X_4 = (x_4, y_4, z_4)\) can be defined at another coordinate system, for example, at system \(X_1 = (x_1, y_1, z_1)\) through a transformation matrix \(a_{14}\) as shown below

\[
X_4 = a_{14}X_1 \tag{2.23}
\]

where the columns of the transformation matrix \(a_{14}\) are the unit vectors of system 1 in system 4.

Using the transformation matrices which are not presented here, the undisturbed incoming wind velocity seen by the blade \(V_0\) can be obtained through the transformation from the natural (fixed) coordinate system 1 where the incoming wind velocity is given as \(V_1\) to the blade coordinate system 4:

\[
V_0 = a_{14}V_1 \tag{2.24}
\]

2.3.3 Wind model

In order to be able to calculate the time loads on a turbine caused by wind realistically, the wind field must be modeled with proper spatial and temporal variation. The wind field seen by rotor varies in space and time due to atmospheric turbulence, tower shadow and wind shear. In order to take the unsteadiness in the wind field due to these factors into account in the aeroelastic code, the following implementations are performed.
2.3.3.1 Atmospheric Turbulence

The variation in the incoming wind field seen by the rotor due to atmospheric turbulence is obtained using an in-house code implementation of Mann model provided from the Fluid Mechanics section at DTU Mechanical Engineering Department. Using this code provided, the wind seen by rotor $V_0(x, y, z, t)$ is computed in a polar grid which spans through the area swept by rotor at each radial and azimuthal angle node as shown in Figure 2.5.

![Figure 2.5: Polar grid of the wind model, showing the grid points swept by the blade](image)

The necessary input for the wind simulation are the radius of the rotor, average wind speed, turbulence intensity, the location of radial nodes and number of azimuthal angles on the full circle, time step size and number of time steps. Once the time series of incoming wind for each point on the grid is obtained as wind input using the wind model, first the data had to manipulated in order to be used for interpolation in Matlab and then an implementation for interpolating $V_0$ from this wind speed input at each time step, radial point on the blade and the azimuthal angle is made in the aeroelastic code.

The standard turbulent wind input for the code verification test of the aeroelastic code were $V_{hub} = 11.4$ and $V_{hub} = 18$ m/s average wind speed at hub height with a turbulence intensity ($I_{ref}$) of 0.14. All turbulent wind inputs including these standard inputs for verification of the aeroelastic code against OC3 results are obtained through this model. Figure 2.6 shows the time series of turbulent wind input $V_0$ for $V_{hub} = 11.4$ m/s with $I_{ref} = 0.14$ at the center of the grid and at the tip of the blade as an example.

2.3.3.2 Wind Shear Effect

The time averaged boundary layer (Figure 2.7) is given by
CHAPTER 2. AERO-ELASTIC MODELING

\[ V_0(x) = V_0(H) \left( \frac{x}{H} \right)^\nu \]  \hspace{1cm} (2.25)

where $H$ is the hub height, $x$ is the distance from the surface and $\nu$ is the shear parameter which varies between 0.1 and 0.25. Implementing this boundary layer equation in the code enables to include the changes in the wind due to the time variation of distance between each radial location and the sea surface as the turbine rotates.

**Figure 2.6:** Time series of wind speed computed at $r=0$ m and $r=63$ m, for $0^\circ$

**Figure 2.7:** Wind shear model

### 2.3.3.3 Tower Shadow Effect

Tower shadow effect is implemented through a simple model for flow around cylinder as shown in Figure 2.8 which assumes potential flow around cylinder. The velocity
components $V_\theta$ and $V_r$ are determined by the following equations:

$$V_r = V_0 \left( 1 - \left( \frac{a}{R} \right)^2 \right) \cos \theta$$  \hspace{1cm} (2.26)

$$V_\theta = -V_0 \left( 1 + \left( \frac{a}{R} \right)^2 \right) \sin \theta$$  \hspace{1cm} (2.27)

The idea behind tower shadow model is that at every rotation of the blades, they will pass from the front of the tower and the incoming wind velocity when the blade is passing near the tower would be disturbed as shown in Figure 2.8 having two components rather than coming from only in z direction. This will change the relative velocity seen at a point on blade as it passes down the tower and therefore will induce fluctuations in the aerodynamic loads on the turbine.

![Figure 2.8: Tower effect model](image)

The effect of tower on the wind seen by the blade during a full rotation of the rotor is demonstrated at three different radial nodes in Figure 2.9; one node is chosen close to blade root, one node in the middle of the blade and the last one close to the tip of the blade. It can be seen from the three figures that $z$ component of the wind velocity rapidly drops as it enters the close vicinity of the tower reaching to its minimum at $\theta_{wing} = 180^\circ$ while a small $y$ component appears in either $-y$ or $y$ direction depending on which side of the tower the blades passes through. It is also seen that at $\theta_{wing} = 180^\circ$ the $y$ component of wind velocity is zero.

### 2.3.4 Prandtl’s Tip Loss Correction Factor

To correct the assumption of infinite number of blades in the classical beam element momentum method, Prandtl’s tip loss correction factor, $F$ is needed. This factor introduces the losses due to finite number of blades and is defined with the following equations:

$$f = \frac{B \cdot R - r_i}{2 \cdot r_i \cdot \sin(\phi)}$$  \hspace{1cm} (2.28)

$$F = \frac{2}{\pi} \arccos \left( e^{-f} \right)$$  \hspace{1cm} (2.29)

Numerically, when the flow angle $\phi$ is zero, $F$ has to be calculated in a different way due to the singularity in the definition of $f$. Therefore, $F$ is made equal to 1 when flow angle is close to zero, i.e. $\sin(\phi) < 0.01$. 
2.3.5 Glauert Correction

Glauert correction is needed when the axial induction factor becomes larger than around 0.4 since the simple momentum theory does not apply any longer for axial induction values larger than 0.4. Glauert correction, \( f_g \) is an empirical relationship between the thrust coefficient \( C_T \) and the axial induction factor \( a \) when the wake is in turbulent state and can be computed as

\[
    f_g = \begin{cases} 
        \frac{a_c}{a} & a \leq a_c \\
        \frac{1}{a} \left( 2 - \frac{a_c}{a} \right) & a > a_c 
    \end{cases}
\]  

(2.30)

2.3.6 Dynamic Wake Model

In order to take into account the delay in the time behavior of aeroelastic loads and power when there is a change in thrust, a dynamic inflow model must be applied. For a pitch regulated wind turbine, as is in this project, the thrust is changing each time the blades are pitched. Therefore, it is of high importance for a pitch regulated turbine to have a dynamic inflow model to capture the time response of power and aeroelastic loads.
2.3. CALCULATION OF WIND LOADS

realistically. The dynamic inflow model used in this project is the dynamic wake model by Øye. This model acts as a dynamic filter for the induced velocities using two first order differential equations:

\[ W_{\text{int}} + \tau_1 \frac{dW_{\text{int}}}{dt} = W_{qs} + k\tau_1 \frac{dW_{qs}}{dt} \]  

(2.31)

\[ W + \tau_2 \frac{dW}{dt} = W_{\text{int}} \]  

(2.32)

\( W_{qs} \) is the quasi-static value found by equations 2.21 and 2.22, \( W_{\text{int}} \) is an intermediate value and \( W \) is the final filtered value of the induced velocity.

2.3.7 Dynamic Stall Model

Dynamic stall model is needed in order to take into account the time delay that occurs on the loads when the flow angle of attack changes over time due to wind shear, atmospheric turbulence, tower passage etc. It is shown that if a dynamic stall model is not used, the aeroelastic model might compute non-existing flapwise vibrations \[14\]. Therefore, it is recommended to use a dynamic stall model in calculations of lift for stability reasons. In this project the trailing edge stall model made by Øye is implemented. Trailing edge stall means that separation starts at the trailing edge and increases gradually upstream as the angle of attack increases. This dynamic stall is modeled through a separation function \( f_s \) which defines the degree of stall in the following expression for lift coefficient:

\[ C_l = f_s C_{l,\text{inv}}(\alpha) + (1 - f_s) C_{l,fs}(\alpha) \]  

(2.33)

In equation 2.33, lift coefficient is defined by partial contributions of the lift coefficient for inviscid flow \( C_{l,\text{inv}}(\alpha) \) and the lift coefficient for fully separated flow \( C_{l,fs}(\alpha) \).

The \( f_s \) value in equation 2.33 is obtained through the following relations

\[ \frac{df_s}{dt} = f_s^{st} - f_s \]  

(2.34)

In the above expression, \( f_s^{st} \) denotes the value of \( f_s \) that produces the static airfoil data when used in equation 2.33 and \( f_s \) is assumed to always try to go back to this static value, \( f_s^{st} \) as given in 2.34. Integrating analytically the equation 2.34, \( f_s \) can be calculated at each time step as:

\[ f_s(t + \delta t) = f_s^{st} + (f_s(t) - f_s^{st}) \exp(-\frac{\delta t}{\tau}) \]  

(2.35)

In the above equations 2.34 and 2.35, \( \tau \) is a time constant is approximately equal to \( \frac{Ac}{V_{rel}} \) where \( c \) is the local chord, \( V_{rel} \) is the relative velocity seen by the blade and \( A \) is a constant that takes a value of about 4.
The implementation of the dynamic stall model involved obtaining dynamic airfoil data given by equation 2.33 from the static airfoil data provided. In order to derive the dynamic lift coefficient using the given expressions above, first the curves for $C_{l,fs}(\alpha)$ and $C_{l,inv}(\alpha)$ had to be defined through analytical relations. $C_{l,inv}(\alpha)$ is for flow without any separation and is normally the extrapolation of the static airfoil data in the linear region. To obtain $C_{l,fs}(\alpha)$, equation 2.36 is used and then fitted such that it passes through the starting point of the linear region. A demonstration of this method can be seen for the static airfoil data provided for the radial node at 48.65 m in Figure 2.10.

\[
C_{l,fs} = K \cos(\alpha) \sin(\alpha) \quad (2.36)
\]

Figure 2.10: Curves for static, inviscid and fully separated lift coefficients for the airfoil data NACA64 A17.dat

Obtaining the curves for $C_{l,fs}$ and $C_{l,inv}$ for each airfoil data with the method demonstrated above, $f_{s}^{st}$ values that reproduce the static airfoil data are computed for each angle of attack $\alpha$ using the relation given in equation 2.33.

### 2.3.8 Load distribution and Thrust, Moment and Power Calculation

Once the unsteady BEM algorithm is applied, the tangential and normal distribution of the loads acting on each blade is known. To obtain the global parameters, power and thrust, these distributions have to integrated over the length of the blade.

The tangential and normal loads on each blade at each radial position at each time step is computed as shown in the following equations using the relative velocity and angle of attack values.

\[
L = \frac{1}{2} \rho |v_{rel}|^2 c C_{l}(\alpha) \quad (2.37)
\]
2.4 Calculation of Wave Loads

Since this project deals with an offshore wind turbine the time-dependent wave loads have to be included to the external loads acting on the turbine as well as the wind loads. The wave loads are provided to the aeroelastic model as a predefined external input, i.e. they are independent from the wind loads and the structural response of the turbine. Wind loads acting on the turbine structure at each time step, however, depends on the wave loads since aerodynamic forces depend on the vibrating velocity of the tower structure that occur due to the wave loads. The wave force input is calculated from the wave spectra output of the wave model (Chapter 1) at the specified locations using Morison’s equation. The wave force per length acting on a single node on tower length is given by the Morison equation as:

\[ F = \frac{1}{2} \rho C_D DU |U| + \rho C_M A \dot{U} \] (2.41)

where \( U \) is the wave velocity component, \( \rho \) is the density of water, \( C_D \) is the drag coefficient, \( D \) is the diameter of the monopile, \( C_M \) is the inertia coefficient and \( A \) is the cross-sectional area of the monopile. The first term on the right-hand side is the drag force and the second term is the inertia force [9].

In order to be able to use Morison equation to calculate wave loads, first the time series for wave velocity and water elevation is computed from the wave spectra obtained at a desired location. This is done by using the Matlab routine provided by Henrik Bredmøse which works like an inverse Fourier Transform. The details of the code are outside the scope of this study, but some brief explanation about the code will be mentioned here. The code uses the wave energy spectra data which covers the whole wave simulation period as input file and choses the spectrum occurring at the time step when the wave height is maximum as the “selected wave climate for force calculation” (Figure 2.11).

Choosing the water depth (which is assumed to be 20 m for the standard offshore NREL monopile-tower structure) and the desired length of time and time step size, the routine generates time series for surface elevation \( \eta \), \( u \) and \( v \) components of wave velocity and the time derivatives of wave velocity at a number of elevations from the selected
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Figure 2.11: Time series of significant wave height for waves at FINO1 location obtained by the inverse FFT code

Figure 2.12: Energy spectrum of the selected wave climate from FINO1 data

wave climate spectrum (Figure 2.12). The generated time series for \( \eta \) can be seen in Figure 2.13.

Finally, it computes the time series of the force distribution in both y and z directions at each node along the water depth based on Morison’s equation. Before using this time series forces obtained from this routine in the aeroelastic code, they need to be projected onto the nodes that are defined for the tower modal shape function so that the contribution of wave forces on the generalized forces can be computed.

2.5 Control model

In this project the baseline control system for conventional variable-speed, variable-pitch configuration described in NREL report [13] is implemented. The aim of this control
2.5. CONTROL MODEL

Figure 2.13: Time series of water elevation $\eta$ for the selected climate from FINO1 data

system is to control power-production operation of the turbine. The system consists of two basic control systems which are designed to work independently: A generator torque controller and a blade-pitch controller. The first one aims to maximize the power capture to regulate the generator speed above the rated operating point.

It is important to note that the control system only aims to control power-production operation, therefore additional non-power production operations such as control of start-up and shut down sequences are not implemented here.

Both generator-torque and blade-pitch controllers use the generator speed measurement as the only feedback input. To mitigate high-frequency excitation of the control systems, the input of measured generator speed is filtered before being used in both controllers.

Next section briefly explains the measurement filter and after that, implementations of the two controllers are described in the following sections. All the equations shown are taken from the NREL report [13].

2.5.1 Measurement Filter

The necessary filtering of the input measurement signal is done by using a recursive, single-pole low-pass filter with exponential smoothing. The discrete-time difference equation of the filter is given as

$$y[n] = (1 - \alpha)u[n] + \alpha y[n - 1]$$  \hspace{1cm} (2.42)

$$\alpha = e^{(-2\pi f_c T_s)}$$  \hspace{1cm} (2.43)

Here, $y$ is the filtered generator speed (output), $u$ is the unfiltered generator speed (input), $\alpha$ is the low-pass filter coefficient, $n$ is the discrete-time step index, $T_s$ is the discrete time step and $f_c$ is the corner frequency which is set to be 0.25 Hz.
2.5.2 Baseline Generator Torque Controller

The purpose of the basic torque controller is to maximize the power output from the turbine in the below rated region. In other words, the power coefficient $C_p$ needs to be maximized at each operational point of the turbine in the below rated range. Since power coefficient $C_p$ is a function of the tip speed ratio $\lambda$ and the pitch angle $\theta_p$, and variable speed-pitch regulated rotor is used, it is possible to run the turbine at the optimum operating point at $\lambda_{opt}$ and $\theta_{p,opt}$. This optimum point will enable operation at the maximum power coefficient $C_{p,max}$, therefore maximizing the power output from the turbine.

![Optimum operation curve for a variable speed generator](image)

**Figure 2.14:** Optimum operation curve for a variable speed generator

Figure 2.14 shows the power curves and the optimum curve obtained for a set of wind speeds between 8 and 12 m/s at a pitch angle of $0^\circ$ by using a steady BEM algorithm. The power curves are also presented in terms of power coefficient as a function of tip speed ratio in Figure 2.15.

From such a $C_p$ vs. $\lambda$ curve at the optimum pitch angle $\theta_{p,opt}$, the optimum curve, power can be expressed as a function of the rotational speed for a number of given wind speeds $V_0$ through these relations:

$$ P = \frac{1}{2} \rho V_0^3 A C_p(\lambda) $$

(2.44)

$$ \omega = \frac{\lambda V_0}{R} $$

(2.45)

Using the equations 2.44 and 2.45, rotor torque $M$ at the optimum point $\lambda_{opt}$, $\theta_{opt}$ can be obtained through the relation $P = M \omega$ as

$$ M_{opt} = \frac{1}{2} \rho \omega^2 R^3 A C_{p,max}(\lambda_{opt}, \theta_{opt})/\lambda_{opt}^3 = \text{const} \cdot \omega^2 $$

(2.46)
2.5. CONTROL MODEL

Equation 2.46 shows that generator torque will be proportional to the square of the filtered generator speed in the optimized power capture control region. However, the controller will not control the generator torque by this proportionality throughout the whole below rated region. Instead, the generator torque as function of filtered generator speed is divided into five control regions: 1, 1½, 2, 2½ and 3.

![Power coefficient $C_p$ as a function of tip speed ratio $\lambda$](image)

**Figure 2.15:** Power coefficient $C_p$ as a function of tip speed ratio $\lambda$

![Torque vs. speed diagram showing the control regions](image)

**Figure 2.16:** Torque vs. speed diagram showing the control regions, taken from NREL [13]

Figure 2.16 shows the five control regions of generator torque as a tabulated function of the filtered generator speed. The control regions are described as follows

- **Region 1** is the control region before cut-in wind speed. No power is extracted from the turbine. Instead, the wind is used to accelerate the rotor for start-up.

- **Region 1½** is the linear transitional start-up region.
• **Region 2** is the control region for optimizing power output from the turbine given by equation 2.46.

• **Region 2 ½** is needed to limit tip speed at rated power.

• **Region 3** is the above rated generator speed region. There, power is held constant so the torque is inversely proportional to the filtered generator speed.

![Torque vs. speed response of the variable speed torque controller](image)

**Figure 2.17:** Torque vs. speed response of the variable speed torque controller

Revisiting Figure 2.15 the peak power coefficient can be read as 0.52 at a tip speed ratio of about 9.25. These values are slightly higher than the optimum point obtained in NREL report with $C_{p,\text{max}}$ of 0.482 occurring at tip speed ratio of 7.55 and pitch angle of $0^\circ$.

The differences in the values are due to the difference in the aerodynamic load algorithms. It can be seen from the comparison that if the values obtained by simple steady BEM algorithm are chosen as the optimum operating point for maximizing power output, the tip speed ratio will be too high. Therefore, the operating curve in below rated region is chosen to follow a slightly lower $C_p$ to limit the tip speed ratio. The convenient choice was to follow a $C_p$ of about 0.51 which enabled to limit the tip speed ratio to 7.8 and matched well with the optimum curve of NREL with an optimal constant of proportionality around $0.0256 \text{ Nm/rpm}^2$. The resulting generator torque vs. generator speed response can be seen in Figure 2.17 where the operation curve of the generator and the optimal curve can be observed.

### 2.5.3 Baseline Blade Pitch Controller

In the above rated region (Region 3 in Figure 2.16), blade-pitch control system takes over. In this region the power is held constant at rated power, therefore the aim is to regulate the generator rotational speed only.
Pitch controller is designed using gain-scheduled proportional-integral (PI) control on the speed error between the filtered generator speed and the rated generator speed. The design is based on a single degree of freedom model of the wind turbine where the single DOF is the angular rotation of the shaft. To explain how the control gains are computed, it is important to examine the equation of motion of this 1DOF model.

\[
T_{aero} - N_{gear} T_{gen} = (I_{rotor} + N_{gear}^2 I_{gen}) \frac{d}{dt}(\Omega_0 + \Delta \Omega) = I_{drivetrain} \Delta \ddot{\Omega}
\] (2.47)

where \(T_{aero}\) is the low-speed shaft aerodynamic torque, \(T_{gen}\) is the high-speed shaft generator torque, \(N_{gear}\) is the high-speed to low speed gearbox ratio, \(I_{drivetrain}\) is the drivetrain inertia cast to the low-speed shaft, \(I_{rotor}\) is the rotor inertia, \(I_{gen}\) is the generator inertia relative to the high-speed shaft, \(\Omega_0\) is the rated low-speed shaft rotational speed, \(\Delta \Omega\) is the small perturbation of low-speed shaft rotational speed about the rated speed, \(\Delta \dot{\Omega}\) is the low-speed shaft rotational acceleration and \(t\) is the simulation time.

For this 1 DOF turbine model, the small perturbations of the pitch angles about their operating point \(\Delta \theta\) is derived for PID control in the NREL report as

\[
\Delta \theta = K_p N_{gear} \Delta \Omega + K_I \int_0^t N_{gear} \Delta \Omega \, dt + K_D N_{gear} \Delta \dot{\Omega}
\] (2.48)

where \(K_p, K_I,\) and \(K_D\) are the proportional, integral and derivative control gains for the pitch controller, respectively.

The pitch sensitivity \(\frac{\partial P}{\partial \theta}\) is an aerodynamic property of the rotor and depends on the wind speed, rotor speed and pitch angle, i.e. \(\frac{\partial P}{\partial \theta} = f(V_0, \Omega, \theta)\). To find the pitch sensitivity, unsteady BEM routine is run at the rated rotor speed at a number of given steady and uniform wind speeds and the corresponding pitch angles that produce the rated mechanical power. Then, at each of these operating points the pitch angle is perturbed and the resulting variation in the aerodynamic power is computed using numerical differentiation. Power versus pitch angle and the pitch angles corresponding to the rated power operation for the above rated wind speed range can be seen in Figure 2.18.

The pitch angles for obtaining rated power at each wind speed and the corresponding pitch sensitivities are presented in Table 2.2.

It is shown in the NREL report that pitch sensitivity varies considerably in the above rated region, therefore constant pitch sensitivity and constant PI gains are not adequate. However, it is also shown that the pitch sensitivity varies linearly with the pitch angle. Using this linear relationship a simple technique for gain schedule implementation based on pitch angle is presented as

\[
K_p = \frac{2 I_{drivetrain} \Omega_0 \omega_{q_{\text{nom}}}^2}{N_{gear} \left(- \frac{\partial P}{\partial \theta} (\theta = 0) \right)} \cdot GK(\theta)
\] (2.49)

\[
K_I = \frac{I_{drivetrain} \Omega_0 \omega_{q_{\text{nom}}}^2}{N_{gear} \left(- \frac{\partial P}{\partial \theta} (\theta = 0) \right)} \cdot GK(\theta)
\] (2.50)
Figure 2.18: Power vs. pitch curves for wind speeds given in Table 2.2 showing the optimum pitch values at rated operating point

<table>
<thead>
<tr>
<th>Wind speed [m/s]</th>
<th>Rotor Speed [rpm]</th>
<th>Pitch Angle [°]</th>
<th>$\frac{\partial p}{\partial \theta}$</th>
<th>$\frac{\partial p}{\partial \theta}$ NREL</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.2 (rated)</td>
<td>12.1</td>
<td>0</td>
<td>-3.37E+07</td>
<td>-2.82E+07</td>
</tr>
<tr>
<td>12</td>
<td>12.1</td>
<td>3.7204</td>
<td>-1.98E+07</td>
<td>-4.37E+07</td>
</tr>
<tr>
<td>13</td>
<td>12.1</td>
<td>6.4406</td>
<td>-3.07E+07</td>
<td>-5.17E+07</td>
</tr>
<tr>
<td>14</td>
<td>12.1</td>
<td>8.5251</td>
<td>-3.85E+07</td>
<td>-5.84E+07</td>
</tr>
<tr>
<td>15</td>
<td>12.1</td>
<td>10.3175</td>
<td>-4.51E+07</td>
<td>-6.44E+07</td>
</tr>
<tr>
<td>16</td>
<td>12.1</td>
<td>11.9363</td>
<td>-5.13E+07</td>
<td>-7.05E+07</td>
</tr>
<tr>
<td>17</td>
<td>12.1</td>
<td>13.4231</td>
<td>-5.73E+07</td>
<td>-7.65E+07</td>
</tr>
<tr>
<td>18</td>
<td>12.1</td>
<td>14.8204</td>
<td>-6.43E+07</td>
<td>-8.39E+07</td>
</tr>
<tr>
<td>19</td>
<td>12.1</td>
<td>16.139</td>
<td>-7.09E+07</td>
<td>-9.07E+07</td>
</tr>
<tr>
<td>20</td>
<td>12.1</td>
<td>17.397</td>
<td>-7.62E+07</td>
<td>-9.47E+07</td>
</tr>
<tr>
<td>21</td>
<td>12.1</td>
<td>18.6135</td>
<td>-8.07E+07</td>
<td>-9.90E+07</td>
</tr>
<tr>
<td>22</td>
<td>12.1</td>
<td>19.7893</td>
<td>-8.57E+07</td>
<td>-1.06E+08</td>
</tr>
<tr>
<td>23</td>
<td>12.1</td>
<td>20.9258</td>
<td>-9.27E+07</td>
<td>-1.14E+08</td>
</tr>
<tr>
<td>24</td>
<td>12.1</td>
<td>22.0228</td>
<td>-9.81E+07</td>
<td>-1.20E+08</td>
</tr>
<tr>
<td>25</td>
<td>12.1</td>
<td>23.0883</td>
<td>-1.06E+08</td>
<td>-1.25E+08</td>
</tr>
</tbody>
</table>

Table 2.2: Sensitivity of aerodynamic power to blade pitch angle in control region 3

where $G_K(\theta)$ is the dimensionless gain-correction factor defined as

$$G_K(\theta) = \frac{1}{1 + \frac{\theta}{\theta_K}} \quad (2.51)$$

The angle $\theta_K$ is the pitch angle at which the pitch sensitivity doubles from the pitch value at the rated operating point. Figure 2.19 shows the linear trend of the pitch sensitivities and the best-line fitted onto these values.

The flowchart of the control system is presented in Appendix C.
2.5. CONTROL MODEL

Figure 2.19: Best fit line of pitch sensitivity in control region 3
Chapter 3

Verification of the aeroelastic code

The aeroelastic model has to be verified before being used in the analysis of combined wind and wave loads case. For this purpose, the model is compared against some of the load case configurations in the OC3 report [5]. These load cases, which are shown in Table 3.1 were defined to compare the different offshore aeroelastic codes that take part in the OC3 project and aimed at identifying model-dependent differences in the simulation results which make up the first project phase. The complete results of this project phase is available in an online database which provided essential resource for verification purposes of the aeroelastic code.

In Phase I a set of 16 load cases has been defined for the offshore wind turbine with monopile substructure and rigid foundation in 20 m water depth. For each load case, a total of 47 sensors, as indicated in figure 2, are analysed. The analyses of the sensors are performed in the time domain for the deterministic load cases and in the frequency domain as well as on basis of statistical parameters and damage equivalent loads for the stochastic load cases. Furthermore, the modal properties in terms of the coupled subsystem eigenfrequencies are analysed.

The individual subsystems are modelled flexible or rigid dependent on the actual load case. In addition, the environmental conditions in terms of the wind and wave field are varied. By this approach the offshore wind turbine model is reduced to the following configurations, which allowed for identification of model-dependent differences in the simulation results:

- Completely rigid structure
- Flexible onshore wind turbine (rigid substructure)
- Flexible offshore structure with tower top mass (rigid nacelle and rotor)
- Fully flexible offshore wind turbine

All relevant aerodynamic and hydrodynamic effects, e.g. turbulence, tower shadow, dynamic stall, wind shear and Wheeler stretching are included in the load cases of Phase I and will be included in investigations of subsequent phases as well. Table 3 provides an overview on the analysed load cases in Phase I. Furthermore, the modal properties in terms of the coupled subsystem eigenfrequencies have been investigated within the load case set 1.x.

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Flexible subsystems</th>
<th>Wind conditions</th>
<th>Wave conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1a</td>
<td>None(^c)</td>
<td>(V_{hub} = 8\text{ m/s: Steady, uniform})</td>
<td>None: (\rho_{water} = 0)</td>
</tr>
<tr>
<td>2.1b</td>
<td>None</td>
<td></td>
<td>None: (\rho_{water} = 0)</td>
</tr>
<tr>
<td>2.2</td>
<td>None</td>
<td>(V_{hub} = 11.4\text{ m/s, } I_{ref} = 0.14)</td>
<td>None: (\rho_{water} = 0)</td>
</tr>
<tr>
<td>2.3</td>
<td>None</td>
<td>(V_{hub} = 18\text{ m/s, } I_{ref} = 0.14)</td>
<td>None: (\rho_{water} = 0)</td>
</tr>
<tr>
<td>2.4</td>
<td>None</td>
<td>None: (\rho_{water} = 0)</td>
<td>Regular Airy, (H = 6\text{ m, } T = 10s)</td>
</tr>
<tr>
<td>2.5</td>
<td>None</td>
<td>None: (\rho_{water} = 0)</td>
<td>Irregular Airy, (H = 6\text{ m, } T = 10s)</td>
</tr>
<tr>
<td>2.6</td>
<td>None</td>
<td>None: (\rho_{water} = 0)</td>
<td>Stream Function, (H = 6\text{ m, } T = 10s)</td>
</tr>
<tr>
<td>3.1</td>
<td>Tower, drivetrain, blades</td>
<td>(V_{hub} = 8\text{ m/s: Steady, uniform})</td>
<td>None: (\rho_{water} = 0)</td>
</tr>
<tr>
<td>3.2</td>
<td>Tower, drivetrain, blades</td>
<td>(V_{hub} = 11.4\text{ m/s, } I_{ref} = 0.14)</td>
<td>None: (\rho_{water} = 0)</td>
</tr>
<tr>
<td>3.3</td>
<td>Tower, drivetrain, blades</td>
<td>(V_{hub} = 18\text{ m/s, } I_{ref} = 0.14)</td>
<td>None: (\rho_{water} = 0)</td>
</tr>
<tr>
<td>4.1</td>
<td>Substructure, tower</td>
<td>None: (\rho_{water} = 0)</td>
<td>Regular Airy, (H = 6\text{ m, } T = 10s)</td>
</tr>
<tr>
<td>4.2</td>
<td>Substructure, tower</td>
<td>None: (\rho_{water} = 0)</td>
<td>Irregular Airy, (H = 6\text{ m, } T = 10s)</td>
</tr>
<tr>
<td>4.3</td>
<td>Substructure, tower</td>
<td>None: (\rho_{water} = 0)</td>
<td>Stream Function, (H = 6\text{ m, } T = 10s)</td>
</tr>
<tr>
<td>5.1</td>
<td>All(^c)</td>
<td>(V_{hub} = 8\text{ m/s: Steady, uniform})</td>
<td>Regular Airy, (H = 6\text{ m, } T = 10s)</td>
</tr>
<tr>
<td>5.2</td>
<td>All(^c)</td>
<td>(V_{hub} = 11.4\text{ m/s, } I_{ref} = 0.14)</td>
<td>Irregular Airy, (H = 6\text{ m, } T = 10s)</td>
</tr>
<tr>
<td>5.3</td>
<td>All(^c)</td>
<td>(V_{hub} = 18\text{ m/s, } I_{ref} = 0.14)</td>
<td>Irregular Airy, (H = 6\text{ m, } T = 10s)</td>
</tr>
</tbody>
</table>

\(^c\) The subsystems included in the offshore wind turbine model are the substructure, tower, drivetrain & blades
\(^d\) The support structure consists of the foundation, substructure and tower. The foundation in Phase I is rigid.

Figure 3.1: Summary of load cases and environmental conditions for OC3 Phase 1

It is presented in the OC3 report that the codes compare quite well with each other with some minor differences which are explained by the variety of aerodynamic models and corrections implemented into the individual codes. Since the codes are comparing
well, the aeroelastic code results are presented against one of the existing codes (HAWC2 by Risø) instead of using several of them for the sake of visual simplicity.

The verification is performed for the selected load cases against HAWC2 results in three steps. First, the code is tested against the cases of wind loads only (Cases 2.1-2.3 and 3.1-3.3), then it is tested for the cases of wave loads alone (Cases 2.3-2.4 and 4.1-4.2). These first two steps consist of comparison between the rigid turbine and flexible offshore turbine under each of the specified environmental conditions, i.e. wind alone or waves alone. Finally, the combined wind and wave loads cases (Cases 5.1-5.3) are compared against the HAWC2 results.

3.1 Wind Loads

This section presents comparison of rigid turbine (cases 2.x) and flexible turbine (cases 3.x) results under the effect of wind loads only. The specified wind conditions in OC3 code comparison are

- steady uniform wind with $V_{hub} = 8 \text{ m/s}$
- turbulent wind with $V_{hub} = 11.4 \text{ m/s}$ and $I_{ref} = 0.14$
- turbulent wind with $V_{hub} = 18 \text{ m/s}$ and $I_{ref} = 0.14$

3.1.1 Steady uniform wind

Case 2.1a deals with the simplest case of the aeroelastic model, i.e. rigid turbine under steady and uniform wind speed of 8 m/s with constant rotor speed and pitch. Case 2.1b differs from 2.1a only with a variable rotor speed. Since there is no structural response involved, these two cases provide verification of the implemented aerodynamic model for calculating wind loads (unsteady BEM) without any coupling with the motion of the turbine. Case 3.1 on the other hand has turbine structure dynamics coupled with the aerodynamic loads. Therefore, the effect of response of tower top due to thrust force is involved in the results. The results of power and thrust against HAWC2 results can be seen in Figures 3.3 and 3.2 for cases 2.1a, 2.1b and 3.1. It can be seen from the figures that the results agree well with the OC3 codes for steady uniform wind speed below the rated speed.
## 3.1. WIND LOADS

### Figure 3.2: Time series of thrust on yaw bearing level for load cases 2.1a, 2.1b and 3.1

![Thrust vs Time](image)

### Figure 3.3: Time series of power for load cases 2.1a, 2.1b and 3.1

![Mechanical Power vs Time](image)

![Generator Power vs Time](image)
3.1.2 Turbulent wind, rated speed

In this section same comparison between rigid and flexible turbine is performed for turbulent wind conditions. Cases 2.2 and 3.2 demonstrate the rigid and flexible turbine response under turbulent wind speed at rated value of $V_{hub} = 11.4$. Figures 3.4 and 3.5 show power spectra of generator power and thrust force at yaw bearing level for cases 2.2 and 3.2, which agree well with the HAWC2 results.

![Figure 3.4: Power spectra of the generator power for load cases 2.2 and 3.2](image)

![Figure 3.5: Power spectra of thrust force on yaw bearing level for load cases 2.2 and 3.2](image)

In addition to the power spectra, the time response of case 3.2 can be seen in Figure 3.6.

3.1.3 Turbulent wind, above rated speed

Cases 2.3 and 3.3 represent the load case of turbulent wind with average of 18 m/s at hub height, which is above the rated wind speed. Comparing the generator power for rigid and flexible turbine under this loading case in Figure 3.7 it seen that there is an energy drop at higher frequencies for the flexible turbine. The comparison of thrust force...
Figure 3.6: Time series responses for load case 3.2
CHAPTER 3. VERIFICATION OF THE AEROELASTIC CODE

shown in Figure 3.8 on the other hand shows good agreement with HAWC2 spectra. In addition to the power spectra, the time response of case 3.3 can be seen in Figure 3.9.

*Figure 3.7:* Power spectra of the generator power for load cases 2.3 and 3.3

*Figure 3.8:* Power spectra of the thrust force on yaw bearing level for load cases 2.3 and 3.3

3.2 Wave Loads

The following cases test the aeroelastic code under the effect of wave loads only, therefore they do not involve any aeroelastic response, i.e. no aerodynamic effects. These cases simply demonstrate the loads and response of the tower, which was modeled as an inverted pendulum with a concentrated top mass, under cyclic wave loads. For this purpose two standard wave input is used in the OC3 code comparison:

- Regular (linear) wave with wave height, $H = 6m$ and period, $T = 10s$
- Irregular wave with significant wave height $H_s = 6m$ and peak period $T_p = 10s$
Figure 3.9: Time series responses for load case 3.3
### 3.2.1 Regular Waves

Cases 2.4 and 4.1 deal with the rigid and flexible turbine under regular wave loads, respectively. The comparisons of base shear and overturning moment for both cases are presented in Figures 3.10 and 3.11.

![Figure 3.10: Time series of base shear for load cases 2.4 and 4.1](image1)

![Figure 3.11: Time series of overturning moment for load cases 2.4 and 4.1](image2)

It can be seen from Figure 3.10 that base shear forces compare well for both rigid and flexible turbine. The difference in the flexible turbine comes from the addition of inertia force due to tower vibrations. Base shear comparisons provide a good check for whether the load distribution on the turbine matches with OC3 codes. The overturning moment computation of the aeroelastic model differs slightly from the HAWC2 computations. The reason for that can be explained by the different structural models assumed for the tower. The tower degree of freedom is defined by assuming a single cantilever beam with a top mass throughout the whole monopile and tower structure in my model, whereas for the OC3 codes more complicated models exist. This would make a difference in the stiffness ($EI$) distribution and curvature($\kappa$) functions needed for calculating the
moment distribution along the support structure and therefore would result in differences from the OC3 moments although the external force distributions seem to be agreeing as seen in Figure 3.10.

### 3.2.2 Irregular Waves

When the turbine is subject to irregular waves the following results presented in Figures 3.12 and 3.13 are obtained for base shear and overturning moment.

![Figure 3.12: Power spectra of base shear for load cases 2.5 and 4.2](image)

![Figure 3.13: Power spectra of overturning moment for load cases 2.5 and 4.2](image)

All the figures show a mismatch with the HAWC2 results in higher frequencies although the peaks at the peak wave frequency of 0.1 simulated with good agreement. It is seen from Figures 3.12(a) and 3.13(a) that for the rigid turbine the peak around 0.1 Hz up to 0.7 Hz is matching well with the OC3 codes for both base shear and overturning moment. However, for the case where tower dynamics are involved the code shows less agreement (Figures 3.12(b) and 3.13(b)). One reason can be the differences in the inertial loads due to differences in the structural model of the tower, since the code seems to damping more energy than the HAWC2 results as seen in Figures 3.12(b) and...
Another reason can be the differences in the irregular wave inputs. Although both codes use Pierson-Moskovitz spectrum as the irregular wave, there might be some differences due to numerical implementations.

### 3.3 Combined Wind and Wave Loads

The last step of the verification cases is the combined wind and wave loading cases. In this section the load cases 5.x, where the external loads consist of the combination of the separate wind and wave conditions investigated previously, are presented. These combinations are

- **Case 5.1**: steady uniform wind \( V_{hub} = 8 \, \text{m/s} \), regular wave \( H = 6 \, \text{m}, \, T = 10 \, \text{s} \)
- **Case 5.1**: turbulent wind \( V_{hub} = 11.4 \, \text{m/s} \) and \( I_{ref} = 0.14 \), irregular wave \( H_s = 6 \, \text{m}, \, T_p = 10 \, \text{s} \)
- **Case 5.1**: turbulent wind \( V_{hub} = 18 \, \text{m/s} \) and \( I_{ref} = 0.14 \), irregular wave \( H_s = 6 \, \text{m}, \, T_p = 10 \, \text{s} \)

#### 3.3.1 Steady Uniform Wind, Regular Waves

The first case (case 5.1) is to demonstrate the simple combined case where steady and uniform wind speed and regular wave are combined. It is the combination of load cases for case 3.1 and case 4.1. The comparison of the combined case with these separate load cases is presented in Figure 3.14 for tower top deflection. It can be seen from the Figure 3.14(a) that deflections due to the aerodynamic loading are predicted with an error of about 20% although the behavior of the response is modeled in an accurate way compared to the HAWC2 response. Here, the fluctuations occur at a frequency of around 0.47 Hz which correspond to the 3P frequency for the rotational speed around 9.3 rpm. From Figure 3.14(b) and 3.14(c) it is seen that when waves are present tower motion is mostly dominated by the wave frequency around 0.1 Hz and it is also seen that there also exists a higher frequency response which is around the first eigenfrequency of the system (0.28 Hz). The comparison of rotor speed for load case 3.1 and 5.1 in Figure 3.15 show good agreement with OC3 results. It is also seen from Figure 3.15(b) that the effect of combination of wind and wave loads on the rotor speed fluctuations are predicted similar to the OC3 results. Finally, the comparison of power and thrust time series against HAWC2 results are presented in Figure 3.16 which show good agreement except for the larger dips in the thrust force than the HAWC2 values.
3.3. COMBINED WIND AND WAVE LOADS

Figure 3.14: Time series of tower top deflection for cases 3.1, 4.1 and 5.1

Figure 3.15: Time series comparison of rotor speed for load cases 3.1 and 5.1
3.3.2 Turbulent wind (average at rated speed), Irregular Waves

Figure 3.16 and 3.18 shows the case of irregular waves are combined with the turbulent wind with an average speed around the rated speed of 11.4 m/s. It is seen from the time series in Figure 3.18 that there exists an instability in the controller response when the speed is fluctuating around the rated wind speed, meaning that the controller response is fluctuating between the below rated and above rated region. Although the tips in the NREL report [13] are applied to the controller to reduce the dips in the output during transition between these two regions, it is seen that there still remains a fluctuation in the generator torque and pitch angle command. This results in amplified response at the tower top at the first eigenfrequency of the tower. Therefore, the structural response at the rated wind speed region cannot be simulated accurately. Despite this fact that results are not reliable for comparison against HAWC2 results, the power spectra of the power and thrust force is plotted for further investigation. It is seen that the energy contents are not as accurate as the previous cases for wind only.

Figure 3.17: Power spectra of the generator power and thrust force for load cases 5.2
Figure 3.18: Time series responses for load case 5.2
3.3.3 Turbulent wind (average above rated speed), Irregular Waves

When the wind input is in the well above rated region with an average wind speed of 18 m/s the results become comparable against OC3 results as no dips occur in the controller response. This due to the fact that we are well above the rated region therefore the torque and pitch controller do not go through transition regions. The power spectra of generator power and thrust force comparison is presented in Figure 3.19. It is seen that the response in the high frequency has a drop of energy although in the lower frequency region the 3P frequency of 0.6 Hz for a rotor speed around the rated speed (12 rpm) is simulated close to the HAWC2 results. For further investigation time series of the response for case 5.3 is presented in Figure 3.20.

![Figure 3.19: Power spectra of the generator power and thrust force for load cases 5.3](image_url)
Figure 3.20: Time series responses for load case 5.3
Chapter 4

Analysis and Results

4.1 Analysis of wave-wind directionality with standard inputs

After the verification of combined wind and wave case, the code is run for a test of wind and wave misalignment. First, the simplest case with steady uniform wind of 8 m/s and regular waves with $H = 6m$ and $T = 10s$ is investigated for wind and wave misalignment of $90^\circ$. The comparison of the tower top deflection in y and z direction with the deflections when wind and waves are aligned are presented in Figure 4.1. It is seen from the figure that when waves are aligned with wind the tower vibrates with wave frequency and the first eigenfrequency in z direction while y deflections are zero. When the waves are misaligned with $90^\circ$ deflections in the z direction occur at first eigenfrequency and deflections in y direction occur at the wave frequency. The effect of aerodynamic damping when waves and wind are aligned do not occur significantly in this configuration.

Secondly, the code is run for a turbulent wind and irregular wave combination as in case 5.3 and the results for aligned ($0^\circ$) and misaligned ($90^\circ$) combinations are presented in Figure 4.3 and 4.4. In these figures red line represent the rated value for generator power and rotor torque and for tower top deflection it represents the deflections in y-direction. The wind and wave input for this configuration can be seen in Figure 4.2. Comparing the tower top deflections in Figures 4.3 and 4.4 it is seen that deflections in the direction of the wind do not show significant difference between the aligned and misaligned case. It was expected that when the waves are aligned there would be aerodynamic damping present which acts in the favor of reducing the tower vibrations.

4.2 Analysis of turbine response with FINO1 wave input

After testing the code against verification cases presented in OC3 code collaboration, the code is run for a realistic case in the North Sea, German Bight region where the wave modeling study is performed. Using the wave climate obtained from the UKNS wave model at FINO1 location for an intermediate storm event during May 2009 the response of the tower is simulated for a combined wind and wave loading analysis. In the analysis,
Figure 4.1: Time series of deflections for load case 5.1, wind and waves aligned and misaligned

A 10 minute turbulent wind field at an average speed of 20 m/s with turbulence intensity of 9% is assumed as a representative of the offshore wind input during period of highest wave height. Both wind and wave inputs used in the simulation can be seen in Figure 4.5 and 4.6. As seen from Figure 4.6, the wave forces are higher in the direction of the wind. Therefore, it is expected that most of the tower deflection will occur in the direction of the wind.

The results of the simulation are presented in Figure 4.7. The results show that with the given input for wind and waves the power and rotational speed outputs are quite stable around their rated values. The deflections are simulated to be in the order of 20 cm in the main wind direction and about a few centimeters in the lateral direction. It is seen that fluctuations are mainly occurring in the first tower eigenfrequency.
Figure 4.2: Time series of wind speed and water elevation for load case 5.3
Figure 4.3: Time series response for load case 5.3, wind and waves aligned
4.2. ANALYSIS OF TURBINE RESPONSE WITH FINO1 WAVE INPUT

Figure 4.4: Time series response for load case 5.3, wind and waves misaligned
Figure 4.5: Wind and wave input for FINO1 simulation

Figure 4.6: Total wave force in y and z directions at FINO1 station
Figure 4.7: Time series responses for FINO1 simulation
Chapter 5

Conclusion

In this project the response of an offshore wind turbine under combined wind and wave loading is investigated by means of an aeroelastic code for offshore wind turbine. First, a wave modeling task is carried out by MIKE 21 SW spectral wave modeling software in the area of interest (German Bight), in order to obtain the wave conditions around the offshore wind turbine. The existing wave model (UKNS) is validated at FINO1 location with the observations taken at the measurement station. Then, a sensitivity analysis is carried out on the model with respect to wave input, boundary conditions and mesh resolution. It is seen that the model was most sensitive to wind input and the boundary conditions. This was expected since wind is the driving input for wind driven waves and it is important to have the information about swell carried from outside the model domain to predict the wave climate more accurately in the domain.

After the assessment of wave conditions, an aeroelastic code that combines wind and wave input for response under simultaneous loading of both wind and waves is developed. Development of an aeroelastic code for offshore turbine involved a number of challenges such as inverse FFT transformation of wave input, modeling the wind field, implementation of a control model and several correction models in aerodynamic model and modeling the structure of the tower. A lot of effort has been made in all these tasks both in theoretical and numerical aspects among which the most challenging ones were related with numerical implementation of control model and structural model of the turbine. Considerable amount of time has been spent on the modules of unsteady BEM algorithm and the pitch controller implementation.

Besides assessment of wave inputs in the aeroelastic code, another important point in developing aeroelastic codes for offshore wind turbines is the structural model of the turbine. In this project, a simple model approach is employed, with few degrees of freedom. Comparing the results of the code with HAWC2 results showed that aerodynamic model alone was in good agreement, however for the cases where the structural response is coupled, some predictions were not as expected. It was seen that the combined wind and wave case for steady uniform wind and regular wave were simulated with good agreement in the behavior and with an error around 20% in the magnitude. However, for the cases with turbulent wind and irregular waves, the use of the code turned out to be
somewhat limited. One reason for this is the differences in the control model, where a simple model is assumed in the project, whereas the OC3 codes have more complicated models that help getting more stable results. Another reason is the structural model, i.e. the degrees of freedom and the equation of motion. It is known that the OC3 codes use much more complicated degrees of freedom and therefore end up in more accurate predictions of the tower vibrations.

The aeroelastic code is run for a realistic extreme case in North Sea where the wave conditions are modeled and analyzed. This provided a simple simulation of the combined wind and wave loads for an offshore wind turbine in the area of interest. These results provide an estimation of the deflections and loads for an NREL offshore wind turbine which can be used for an extreme load analysis for design purposes of support structure. These results can be improved with a more stable control system that reduces the loads caused by the controller response down to a more accurate level. It is important to note that the code had limitations for analysis of wind and wave misalignment which is a very important topic that a lot of research has been going on. Further improvements in the code such as increasing the complexity of degrees of freedom and control model can make the code a useful tool for directionality analysis. In addition to an intermediate storm event, a further analysis that can be done with the code is the analysis of a 50-year-event. Other than extreme event analysis, the code can be used further for longer periods of simulations for a fatigue analysis.

Overall the code provided a good means of combining MIKE 21 SW wave model outputs with a storm wind field to investigate the response of an offshore wind turbine at German Bight area in North Sea under these extreme conditions, which compared well with the OC3 offshore aeroelastic code for certain sets of standard inputs.
Appendix A

Definition of quality indices
Quality indices for comparison of observed and modelled data

The model data was compared with observations for the purpose of model validation and assessment of the data accuracy. To obtain an objective and quantitative measure of how well the model data compared to the observed data a number of statistical parameters (quality indices, QI’s) were calculated.

Prior to the comparisons the model data was synchronized to the time step of the observations so that both time series have equal length and overlapping time stamps. For each valid observation, measured at time \( t \), the corresponding model value was found using linear interpolation between the model time steps before and after \( t \).

The comparisons of the synchronized observed and modelled data are illustrated in the following ways:

- Time series plot
- Histogram of occurrence vs. magnitude
- Scatter plot incl. sorted data and QI’s
- Histogram of bias vs. magnitude
- Histogram of bias vs. direction

The quality indices are described below and their definitions are listed in the following table. Most of the quality indices are based on all data points together and hence the quality indices should be considered averaged measures for the entire data set and may not be representative of the accuracy during rare events.

<table>
<thead>
<tr>
<th>Description</th>
<th>Abbreviation and Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of valid observations</td>
<td>( N )</td>
</tr>
<tr>
<td>Mean of observations</td>
<td>( MEAN = \frac{1}{N} \sum_{i=1}^{N} OBS_i )</td>
</tr>
<tr>
<td>Mean of difference</td>
<td>( BIAS = \frac{1}{N} \sum_{i=1}^{N} (OBS - MOD)_i )</td>
</tr>
<tr>
<td>Mean of absolute difference</td>
<td>( AME = \frac{1}{N} \sum_{i=1}^{N}</td>
</tr>
<tr>
<td>Root mean square of difference</td>
<td>( RMSE = \left( \frac{1}{N} \sum_{i=1}^{N} (OBS - MOD)_i^2 \right)^{\frac{1}{2}} )</td>
</tr>
<tr>
<td>Scatter index (unbiased)</td>
<td>( SI_{\text{unbiased}} = \sqrt{\frac{\sum_{i=1}^{N} (OBS - MOD - BIAS)_i^2}{MEAN}} )</td>
</tr>
</tbody>
</table>
Correlation coefficient

\[ CC = \frac{\sum_{i=1}^{N} (OBS_i - MEAN)(MOD_i - \bar{MOD})}{\sqrt{\sum_{i=1}^{N} (OBS_i - MEAN)^2 \sum_{i=1}^{N} (MOD_i - \bar{MOD})^2}} \]

Peak ratio of \( N_{\text{peak}} \) events

\[ PR(N_{\text{peak}}) = \frac{1}{N_{\text{peak}}} \sum_{i=1}^{N_{\text{peak}}} \frac{MOD_i}{OBS_i} \]

Regression line slope and intercept

\( \text{REGS and REGI} \)

The BIAS is the mean difference between the modelled and observed data and AME is the mean of the absolute difference. RMSE is the root mean square of the difference.

The scatter index (SI) is a non-dimensional measure of the difference calculated as the unbiased root-mean-square difference relative to the mean value of the observations. In open water, a SI below 0.2 is usually considered a small difference (excellent agreement) for significant wave heights. In confined areas where mean significant wave heights are generally lower, a slightly higher SI may be acceptable.

The correlation coefficient (CC) is a non-dimensional measure reflecting the degree to which the variation of the first variable is reflected in the variation of the second variable. A value close to 0 indicates very limited or no correlation between the two data sets, while a value close to 1 indicates a very high or perfect correlation. Typically, a CC above 0.9 is considered a high correlation (good agreement) for wave heights.

The peak ratio (PR) is the average of the \( N_{\text{peak}} \) highest model values divided by the average of the \( N_{\text{peak}} \) highest observations. The peaks are found through the peak-over-threshold (POT) method applying an average annual number of exceedance of 4 and an inter event time of 36 hours. A general underestimation of the modelled peak events results in PR below 1, while an overestimation results in a PR above 1.

The regression line slope and intercept (REGS and REGI) are found from a linear fit to the data points in a least square sense. A regression line slope different from 1 may indicate a trend in the difference.
Appendix B

Wind and Wave time series for FINO1, STORM and WATCH data
Figure B.1: Time series comparisons of $U_{10}$ and $U_{10}$.
Figure B.2: Time series comparisons of $H_m0$, $T_p$, $T_02$ and $MWD$, sensitivity with respect to wind input
APPENDIX B. WIND AND WAVE TIME SERIES FOR FINO1, STORM AND WATCH DATA

Figure B.3: Time series comparisons of $H_m^0$, $T_p$, $T_02$ and MWD, sensitivity with respect to boundary conditions.
Figure B.4: Time series comparisons of $H_{m0}$, $T_p$, $T_02$ and $MWD$, sensitivity with respect to mesh resolution
Appendix C

Control system

Figure C.1: Diagram of NREL torque controller and pitch controller [13]
MIKE 21 WAVE MODELLING

MIKE 21 SW - Spectral Waves FM

Short Description
MIKE 21 SW - SPECTRAL WAVE MODEL FM

MIKE 21 SW is a state-of-the-art third generation spectral wind-wave model developed by DHI Water & Environment. The model simulates the growth, decay and transformation of wind-generated waves and swell in offshore and coastal areas.

MIKE 21 SW includes two different formulations:
- Fully spectral formulation
- Directional decoupled parametric formulation

The fully spectral formulation is based on the wave action conservation equation, as described in e.g. Komen et al (1994) and Young (1999). The directional decoupled parametric formulation is based on a parameterisation of the wave action conservation equation. The parameterisation is made in the frequency domain by introducing the zeroth and first moment of the wave action spectrum. The basic conservation equations are formulated in either Cartesian co-ordinates for small-scale applications and polar spherical co-ordinates for large-scale applications.

The fully spectral model includes the following physical phenomena:
- Wave growth by action of wind
- Non-linear wave-wave interaction
- Dissipation due to white-capping
- Dissipation due to bottom friction
- Dissipation due to depth-induced wave breaking
- Refraction and shoaling due to depth variations
- Wave-current interaction
- Effect of time-varying water depth
- Effect of ice coverage on the wave field

The discretisation of the governing equation in geographical and spectral space is performed using cell-centred finite volume method. In the geographical domain, an unstructured mesh technique is used. The time integration is performed using a fractional step approach where a multi-sequence explicit method is applied for the propagation of wave action.

MIKE 21 SW is a state-of-the-art numerical modelling tool for prediction and analysis of wave climates in offshore and coastal areas. © BIOFOTO/Klaus K. Bentzen

A MIKE 21 SW forecast application in the North Sea and Baltic Sea. The chart shows a wave field (from the NSBS model) illustrated by the significant wave height in top of the computational mesh. See also www.waterforecast.com
Computational Features
The main computational features of MIKE 21 SW - Spectral Wave Model FM are as follows:

- Fully spectral and directionally decoupled parametric formulations
- Source functions based on state-of-the-art 3rd generation formulations
- Instationary and quasi-stationary solutions
- Optimal degree of flexibility in describing bathymetry and ambient flow conditions using depth-adaptive and boundary-fitted unstructured mesh
- Coupling with hydrodynamic flow model for modelling of wave-current interaction and time-varying water depth
- Flooding and drying in connection with time-varying water depths
- Cell-centred finite volume technique
- Fractional step time-integration with an multi-sequence explicit method for the propagation
- Extensive range of model output parameters (wave, swell, air-sea interaction parameters, radiation stress tensor, spectra, etc.)

Application Areas
MIKE 21 SW is used for the assessment of wave climates in offshore and coastal areas - in hindcast and forecast mode.

A major application area is the design of offshore, coastal and port structures where accurate assessment of wave loads is of utmost importance to the safe and economic design of these structures.

Measured data are often not available during periods long enough to allow for the establishment of sufficiently accurate estimates of extreme sea states. In this case, the measured data can then be supplemented with hindcast data through the simulation of wave conditions during historical storms using MIKE 21 SW.

Example of a global application of MIKE 21 SW. The upper panel shows the bathymetry. Results from such a model (cf. lower panel) can be used as boundary conditions for regional scale forecast or hindcast models. See http://www.waterforecast.com for more details on regional and global modelling

MIKE 21 SW is particularly applicable for simultaneous wave prediction and analysis on regional scale and local scale. Coarse spatial and temporal resolution is used for the regional part of the mesh and a high-resolution boundary and depth-adaptive mesh is describing the shallow water environment at the coastline.

Example of a computational mesh used for transformation of offshore wave statistics using the directionally decoupled parametric formulation
MIKE 21 SW is also used for the calculation of the sediment transport, which for a large part is determined by wave conditions and associated wave-induced currents. The wave-induced current is generated by the gradients in radiation stresses that occur in the surf zone. MIKE 21 SW can be used to calculate the wave conditions and associated radiation stresses. The long-shore currents and sediment transport are then calculated using the flow and sediment transport models available in the MIKE 21 package. For such type of applications, the directional decoupled parametric formulation of MIKE 21 SW is an excellent compromise between the computational effort and accuracy.
**Model Equations**

In MIKE 21 SW, the wind waves are represented by the wave action density spectrum \( N(\sigma, \theta) \). The independent phase parameters have been chosen as the relative (intrinsic) angular frequency, \( \sigma = 2\pi f \) and the direction of wave propagation, \( \theta \). The relation between the relative angular frequency and the absolute angular frequency, \( \omega \), is given by the linear dispersion relationship

\[
\omega = \sigma \cdot \text{tanh}(kd) = \sigma - k \cdot \vec{U}
\]

where \( g \) is the acceleration of gravity, \( d \) is the water depth and \( \vec{U} \) is the current velocity vector and \( k \) is the wave number vector with magnitude \( k \) and direction \( \theta \). The action density, \( N(\sigma, \theta) \), is related to the energy density \( E(\sigma, \theta) \) by

\[
N = \frac{E}{\sigma}
\]

**Fully Spectral Formulation**

The governing equation in MIKE 21 SW is the wave action balance equation formulated in either Cartesian or spherical co-ordinates. In horizontal Cartesian co-ordinates, the conservation equation for wave action reads

\[
\frac{\partial N}{\partial t} + \nabla \cdot (\vec{F}N) = \frac{S}{\sigma}
\]

where \( N(\vec{x}, \sigma, \theta, t) \) is the action density, \( t \) is the time, \( \vec{x} = (x, y) \) is the Cartesian co-ordinates, \( \vec{F} = (c_x, c_y, c_\sigma, c_\theta) \) is the propagation velocity of a wave group in the four-dimensional phase space \( \vec{x}, \sigma, \theta \)-space. The characteristic propagation speeds are given by the linear kinematic relationships

\[
(c_x, c_y) = \frac{\partial \vec{x}}{\partial t} = \vec{c}_g + \vec{U} = \frac{1}{2} \left( 1 + \frac{2kd}{\sinh(2kd)} \right) \frac{\sigma}{k} + \vec{U}
\]

\[
c_\sigma = \frac{d\sigma}{dt} = \frac{\partial \sigma}{\partial t} + \vec{U} \cdot \nabla \sigma + \frac{k}{2} \cdot \frac{\partial \vec{U}}{\partial x}
\]

\[
c_\theta = \frac{d\theta}{dt} = -\frac{1}{k} \left( \frac{\partial \sigma}{\partial t} \frac{\partial \sigma}{\partial m} + \frac{\partial \vec{U}}{\partial \theta} \right)
\]

Here, \( s \) is the space co-ordinate in wave direction \( \theta \) and \( m \) is a co-ordinate perpendicular to \( s \). \( \nabla_s \) is the two-dimensional differential operator in the \( \vec{x} \)-space.

**Source Functions**

The source function term, \( S \), on the right hand side of the wave action conservation equation is given by

\[
S = S_{in} + S_{nl} + S_{ds} + S_{bot} + S_{surf}
\]

Here \( S_{in} \) represents the momentum transfer of wind energy to wave generation, \( S_{nl} \) the energy transfer due non-linear wave-wave interaction, \( S_{ds} \) the dissipation of wave energy due to white-capping (deep water wave breaking), \( S_{bot} \) the dissipation due to bottom friction and \( S_{surf} \) the dissipation of wave energy due to depth-induced breaking.

The default source functions \( S_{in}, S_{nl} \) and \( S_{ds} \) in MIKE 21 SW are similar to the source functions implemented in the WAM Cycle 4 model, see Komen et al (1994).

The wind input is based on Janssen's (1989, 1991) quasi-linear theory of wind-wave generation, where the momentum transfer from the wind to the sea not only depends on the wind stress, but also the sea state itself. The non-linear energy transfer (through the resonant four-wave interaction) is approximated by the DIA approach, Hasselmann et al (1985). The source function describing the dissipation due to white-capping is based on the theory of Hasselmann (1974) and Janssen (1989). The bottom friction dissipation is modelled using the approach by Johnson and Kofod-Hansen (2000), which depends on the wave and sediment properties. The source function describing the bottom-induced wave breaking is based on the well-proven approach of Battjes and Janssen (1978) and Eldeberky and Battjes (1996).

A detailed description of the various source functions is available in Komen et al (1994) and Sørensen et al (2003), which also includes the references listed above.
**Numerical Methods**

**Directional Decoupled Parametric Formulation**

The directionally decoupled parametric formulation is based on a parameterisation of the wave action conservation equation. Following Holthuijsen et al (1989), the parameterisation is made in the frequency domain by introducing the zeroth and first moment of the wave action spectrum as dependent variables.

A similar formulation is used in the MIKE 21 NSW Near-shore Spectral Wind-Wave Model, which is one of the most popular models for wave transformation in coastal and shallow water environment. However, with MIKE 21 SW it is not necessary to set up a number of different orientated bathymetries to cover varying wind and wave directions.

The parameterisation leads to the following coupled equations

\[
\frac{\partial (m_0)}{\partial t} + \frac{\partial (c_m m_0)}{\partial x} + \frac{\partial (c_m m_0)}{\partial y} + \frac{\partial (c_m m_1)}{\partial \theta} = T_0
\]

\[
\frac{\partial (m_1)}{\partial t} + \frac{\partial (c_m m_1)}{\partial x} + \frac{\partial (c_m m_1)}{\partial y} + \frac{\partial (c_m m_1)}{\partial \theta} = T_1
\]

where \(m_0(x, y, \theta)\) and \(m_1(x, y, \theta)\) are the zeroth and first moment of the action spectrum \(N(x, y, \sigma, \theta)\), respectively. \(T_0(x, y, \theta)\) and \(T_1(x, y, \theta)\) are source functions based on the action spectrum. The moments \(m_n(x, y, \theta)\) are defined as

\[
m_n(x, y, \theta) = \int_0^\infty \omega^n N(x, y, \omega, \theta) d\omega
\]

The source functions \(T_0\) and \(T_1\) take into account the effect of local wind generation (stationary solution mode only) and energy dissipation due to bottom friction and wave breaking. The effects of wave-current interaction are also included. The source functions for the local wind generation are derived from empirical growth relations, see Johnson (1998) for details.

**Space Discretisation**

The discretisation in geographical and spectral space is performed using cell-centred finite volume method. In the geographical domain an unstructured mesh is used. The spatial domain is discretised by subdivision of the continuum into non-overlapping elements. Triangle and quadrilateral shaped polygons are presently supported in MIKE 21 SW. The action density, \(N(\sigma, \theta)\) is represented as a piecewise constant over the elements and stored at the geometric centres.

In frequency space either an equidistant or a logarithmic discretisation is used. In the directional space, an equidistant discretisation is used for both types of models. The action density is represented as piecewise constant over the discrete intervals, \(\Delta\sigma\) and \(\Delta\theta\), in the frequency and directional space.

\[
E(\sigma, \theta) = E(\sigma_{\text{max}}, \theta) \left( \frac{\sigma}{\sigma_{\text{max}}} \right)^{-m}
\]

where \(m\) is a constant (= 5) as proposed by Komen et al (1994).
Integrating the wave action conservation over an area $A_i$, the frequency interval $\Delta\sigma_l$ and the directional interval $\Delta\theta_m$ gives

$$\frac{\partial}{\partial t} \int_{\Delta\theta_n} \int_{\Delta\sigma_l} \int_{A_i} N d\Omega d\sigma d\theta - \int_{\Delta\theta_n} \int_{\Delta\sigma_l} \int_{A_i} \frac{S}{\sigma} d\Omega d\sigma d\theta$$

$$= \int_{\Delta\theta_n} \int_{\Delta\sigma_l} \int_{A_i} \nabla \cdot (\bar{v} N) d\Omega d\sigma d\theta$$

where $\Omega$ is the integration variable defined on $A_i$. Using the divergence theorem and introducing the convective flux $\bar{F} = \bar{v} N$, we obtain

$$\frac{\partial N_{i,l,m}}{\partial t} = -\frac{1}{A_i} \left[ \sum_{p=1}^{NE} (F_n)_{p,j,m} \Delta l_p \right]$$

$$- \frac{1}{\Delta\sigma_l} \left[ (F_\sigma)_{i,l+1/2,m} - (F_\sigma)_{i,l-1/2,m} \right]$$

$$- \frac{1}{\Delta\theta_m} \left[ (F_\theta)_{i,l,m+1/2} - (F_\theta)_{i,l,m-1/2} \right] + \frac{S_{i,l,m}}{\sigma_i}$$

where $NE$ is the total number of edges in the cell, $(F_n)_{p,j,m} = (F_x)_{x,n} + (F_y)_{y,n} - 1$ is the normal flux through the edge $p$ in geographical space with length $\Delta l_p$, $(F_\sigma)_{i,l+1/2,m}$ and $(F_\theta)_{i,l,m+1/2}$ is the flux through the face in the frequency and directional space, respectively.

The convective flux is derived using a first-order upwinding scheme. In that

$$F_n = c_n \left( \frac{1}{2} (N_i + N_j) - \frac{1}{2} c_n (N_i - N_j) \right)$$

where $c_n$ is the propagation speed normal to the element cell face.

**Time Integration**

The integration in time is based on a fractional step approach. Firstly, a propagation step is performed calculating an approximate solution $N^*$ at the new time level $(n+1)$ by solving the homogenous wave action conservation equation, i.e. without the source terms. Secondly, a source terms step is performed calculating the new solution $N^{n+1}$ from the estimated solution taking into account only the effect of the source terms.

The propagation step is carried out by an explicit Euler scheme

$$N_{i,l,m}^{n+1} = N_{i,l,m}^n + \Delta t \left( \frac{\partial N_{i,l,m}}{\partial t} \right)^n$$

To overcome the severe stability restriction, a multi-sequence integration scheme is employed. The maximum allowed time step is increased by employing a sequence of integration steps locally, where the number of steps may vary from point to point.

A source term step is performed using an implicit method (see Komen et al, 1994)

$$N_{i,l,m}^{n+1} = N_{i,l,m}^n + \Delta t \left[ \frac{1-\alpha}{\sigma_i} S_{i,l,m}^* + \frac{\alpha S_{i,l,m}^{n+1}}{\sigma_i} \right]$$

where $\alpha$ is a weighting coefficient that determines the type of finite difference method. Using a Taylor series to approximate $S^{n+1}$ and assuming the off-diagonal terms in $\partial S/\partial \sigma_i = \gamma$ are negligible, this equation can be simplified as

$$N_{i,l,m}^{n+1} = N_{i,l,m}^n + \frac{(S_{i,l,m}^* / \sigma_i) \Delta t}{(1-\alpha \gamma \Delta t)}$$

For growing waves ($\gamma > 0$) an explicit forward difference is used ($\alpha = 0$), while for decaying waves ($\gamma < 0$) an implicit backward difference ($\alpha = 1$) is applied.
**Model Input**

The necessary input data can be divided into following groups:

- **Domain and time parameters**:
  - computational mesh
  - co-ordinate type (Cartesian or spherical)
  - simulation length and overall time step

- **Equations, discretisation and solution technique**:
  - formulation type
  - frequency and directional discretisation
  - number of time step groups
  - number of source time steps

- **Forcing parameters**:
  - water level data
  - current data
  - wind data
  - ice data

- **Source function parameters**:
  - non-linear energy transfer
  - wave breaking (shallow water)
  - bottom friction
  - white capping

- **Initial conditions**:
  - zero-spectrum (cold-start)
  - empirical data
  - data file

- **Boundary conditions**:
  - closed boundaries
  - open boundaries (data format and type)

Providing MIKE 21 SW with a suitable mesh is essential for obtaining reliable results from the model. Setting up the mesh includes the appropriate selection of the area to be modelled, adequate resolution of the bathymetry, flow, wind and wave fields under consideration and definition of codes for essential and land boundaries.

Furthermore, the resolution in the geographical space must also be selected with respect to stability considerations.

As the wind is the main driving force in MIKE 21 SW, accurate hindcast or forecast wind fields are of utmost importance for the wave prediction.

The Mesh Generator is an efficient MIKE Zero tool for the generation and handling of unstructured meshes, including the definition and editing of boundaries.

3D visualisation of a computational mesh

If wind data is not available from an atmospheric meteorological model, the wind fields (e.g. cyclones) can be determined by using the wind-generating programs available in MIKE 21 Toolbox.

The chart shows an example of a wind field covering the North Sea and Baltic Sea as wind speed and wind direction. This is used as input to MIKE 21 SW in forecast and hindcast mode.
Model Output
At each mesh point and for each time step four types of output can be obtained from MIKE 21 SW:

- Integral wave parameters divided into wind sea and swell such as
  - significant wave height, $H_{m0}$
  - peak wave period, $T_p$
  - averaged wave period, $T_{01}$
  - zero-crossing wave period, $T_{02}$
  - wave energy period, $T_{10}$
  - peak wave direction, $\theta_p$
  - mean wave direction, $\theta_m$
  - directional standard deviation, $\sigma$
  - wave height with dir., $H_{m0}\cos\theta_m, H_{m0}\sin\theta_m$
  - radiation stress tensor, $S_{xx}, S_{xy}$ and $S_{yy}$

- Model parameters
  - bottom friction coefficient, $C_f$
  - breaking parameter, $\gamma$
  - Courant number, $Cr$
  - time step factor, $\alpha$
  - characteristic edge length, $\Delta l$
  - area of element, $a$
  - wind friction speed, $u^*$
  - roughness length, $z_0$
  - drag coefficient, $C_D$
  - Charnock parameter, $z_{ch}$

- Directional-frequency wave spectra at selected grid points and or areas as well as direction spectra and frequency spectra

Output from MIKE 21 SW is typically post-processed using the Data Viewer available in the common MIKE Zero shell. The Data Viewer is a tool for analysis and visualisation of unstructured data, e.g. to view meshes, spectra, bathymetries, results files of different format with graphical extraction of time series and line series from plan view and import of graphical overlays.

Various other editors and plot controls in the MIKE Zero Composer (e.g. Time Series Plot, Polar Plot, etc.) can be used for analysis and visualisation.

![Example of model output (directional-frequency wave spectrum) processed using the Polar Plot control in the MIKE Zero Plot Composer](image)

The distinction between wind-sea and swell can be calculated using either a constant threshold frequency or a dynamic threshold frequency with an upper frequency limit.

- Input parameters
  - water level, $h$
  - current velocity, $\overline{U}$
  - wind speed, $U_{10}$
  - wind direction, $\theta_w$
**Validation**

The model has successfully been applied to a number of rather basic idealised situations for which the results can be compared with analytical solutions or information from the literature. The basic tests covered fundamental processes such as wave propagation, depth-induced and current-induced shoaling and refraction, wind-wave generation and dissipation.

Comparison between measured and simulated significant wave height, peak wave period and mean wave period at the Ekofisk offshore platform (water depth 70 m) in the North Sea. (-----) calculations and (o) measurements

A major application area of MIKE 21 SW is in connection with design and maintenance of offshore structures

The model has also been tested in natural geophysical conditions (e.g. in the North Sea, the Danish West Coast and the Baltic Sea), which are more realistic and complicated than the academic test and laboratory tests mentioned above.

Comparison between measured and simulated significant wave height, peak wave period and mean wave period at Fjaltring located at the Danish west coast (water depth 17.5 m). (-----) calculations and (o) measurements
The Fjaltring directional wave rider buoy is located offshore relative to the depicted arrow.

MIKE 21 SW is used for prediction of the wave conditions at the complex Horns Rev (reef) in the southeastern part of the North Sea. At this site, a 168 MW offshore wind farm with 80 turbines has been established in water depths between 6.5 and 13.5 m.

Comparison of frequency spectra at Fjaltring. (▬▬) calculations and (──-) measurements.

The upper panels show the Horns Rev offshore wind farm and MIKE C-map chart. The middle panel shows a close-up of the mesh near the Horns Rev S wave rider buoy (t3, 10 m water depth. The lower panel shows a comparison between measured and simulated significant wave height at Horns Rev S, (▬▬) calculations including tide and surge and (──-) calculations excluding including tide and surge, (o) measurements.
The predicted nearshore wave climate along the island of Hiddensee and the coastline of Zingst located in the microtidal Gellen Bay, Germany have been compared to field measurements (Sørensen et al, 2004) provided by the MORWIN project. From the illustrations it can be seen that the wave conditions are well reproduced both offshore and in more shallow water near the shore. The RMS values (on significant wave height) are less than 0.25m at all five stations.

A MIKE 21 SW hindcast application in the Baltic Sea. The upper chart shows the bathymetry and the middle and lower charts show the computational mesh. The lower chart indicates the location of the measurement stations.
The graphical user interface of the MIKE 21 SW model, including an example of the extensive Online Help system.

**Graphical User Interface**
MIKE 21 SW is operated through a fully Windows integrated Graphical User Interface (GUI). Support is provided at each stage by an Online Help system.

**FEMA Approval of MIKE 21**
The US Federal Emergency Management Agency (FEMA) has, as of May 2001, officially approved MIKE 21 for use in coastal flood insurance studies.

The three modules, which are the hydrodynamic module, near-shore spectral wind wave module and offshore spectral wind wave module, have been accepted for coastal storm surge, coastal wave heights, and coastal wave effect usage.

For more information please check [www.fema.gov](http://www.fema.gov) and [www.dsssoftware.com](http://www.dsssoftware.com).

**Hardware and Operating System Requirements**
The module supports Microsoft Windows XP and Microsoft Windows Vista. Microsoft Internet Explorer 5.0 (or higher) is required for network license management as well as for accessing the Online Help.

The recommended minimum hardware requirements for executing MIKE 21 SW are:

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processor</td>
<td>2 GHz PC (or higher)</td>
</tr>
<tr>
<td>Memory (RAM)</td>
<td>1 GB (or higher)</td>
</tr>
<tr>
<td>Hard disk</td>
<td>40 GB (or higher)</td>
</tr>
<tr>
<td>Monitor</td>
<td>SVGA, resolution 1024x768</td>
</tr>
<tr>
<td>Graphic card</td>
<td>32 MB RAM (or higher), 24 bit true colour</td>
</tr>
<tr>
<td>Media</td>
<td>CD-ROM/DVD drive, 20 x speed (or higher)</td>
</tr>
</tbody>
</table>
**Support**

News about new features, applications, papers, updates, patches, etc. are available here:


For further information on MIKE 21 SW, please contact your local DHI agent or the Software Support Centre:

Software Support Centre  
DHI  
Agern Allé 5  
DK-2970 Hørsholm  
Denmark  
Tel: +45 4516 9333  
Fax: +45 4516 9292  
http://dhigroup.com/Software.aspx  
software@dhigroup.com

**References**


**References on Applications**


*MIKE 21 SW is also applied for wave forecasts in ship route planning and improved service for conventional and fast ferry operators*
Bibliography


[10] MIKE 21 SW Scientific Documentation. DHI.


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