Preface

This report completes the master thesis "Aerodynamic Optimization of Bridge Decks" performed at the department of Mechanics, Fluid Mechanics section, Technical University of Denmark in the period from the 1st of January to the 5th of July the year 2010.

The work is conducted in collaboration with the consultant engineering company COWI that has main office in Kgs. Lyngby. The supervisors are Ass. Prof. Dr. Jens Honore Walther, Ph.D. Student Johannes Tophej Rasmussen both from the section of Fluid Mechanics and Allan Larsen from COWI. Advice and guidance throughout this work given by the supervisors is highly appreciated and acknowledged.

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Abstract

The present work is a study of the influence of when a gap is introduced in the center of a simplified generic geometry resembling a bridge deck according to aeroelastic instability or flutter. The critical wind speed for flutter $U_c$ is used to identify the wind speed, where the structure shows aeroelastic instability. The aerodynamic forces on a body subjected to a free stream are determined using a discrete vortex method implemented in the existing code DVMFLOW. For reason of completeness and comparison two methods for determination of $U_c$ will be used in this study. The first method is to subject the geometry to forced motion, and then use the aerodynamic derivatives extracted from the force signal. The second method is to elastically suspend the geometry and subject it to different incoming velocities and then observing when instability occurs. The variation of $U_c$ is determined with the first method for the range from no gap to a gap size of 7.5 times the thickness of the geometry. The result obtained with no gap, is validated against results found for Great Belt East bridge both from simulations and wind tunnel experiments. A significant initial increase of around 65%, compared to the reference state with no gap, is observed toward the gap of half the thickness. The maximum gain of around 85% is found for a gap of 1.5 the thickness. The second method is slightly more conservative in comparison with the previous initial increase, by only showing a gain of around 40%. A general agreement, of the behavior of $U_c$, between the two methods has been observed. The initial increase in $U_c$ is especially interesting, as it presents a potential for reducing the construction costs, as a small gap limits the amount of concrete used. The increase is believed mainly to be caused by pressure equalization in contrast to the general expectation that aerodynamic damping is the leading effect.
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1 Introduction

The evaluation of the past has shown the consequence of the lack of understanding and knowledge regarding construction of structures such as buildings, bridges and etc. This lack has caused many fatal errors, but it has inspired engineers and researchers to investigate these errors such that they can be avoided in the future. The purpose of the investigations are also to prevent new and unknown problems from arising.

One of the most important incidents for science and engineering was the collapse of the First Tacoma Narrows bridge on November 7 in the year 1940. This was only four months after the bridge had opened to traffic on July 1st the same year. At the time of construction, the First Tacoma Narrows bridge was the third longest suspension bridge in the world. The reason for the collapse was aeroelastic instability also known as flutter.

Aeroelastic instability or flutter is a effect of the coupling between the aerodynamic forces and the dynamics of the structure. The term flutter describes the aeroelastic oscillations of slender structures such as thin airfoils, suspended-span bridge decks, etc. This term has been variously used, however recently in more restricted applications which for wind engineering mainly has been classical flutter and single degree of freedom flutter\cite{7}. Classical flutter is where two degrees of freedom of the structure i.e. vertical translation and rotation couple in an unstable oscillation. Single degree of freedom flutter is torsional instability of a structure that e.g. is undergoing strongly separated flow.

This showed the importance of considering aerodynamics and aeroelastics when constructing suspension bridges. Since then this field has been the subject of a significant amount of research which has influenced the design of the succeeding suspension bridges in the world.

Recent experience has identified a new issue that needs to be addressed when constructing cable supported suspension bridges. Trapezoidal box girder decks is a typical choice for cable supported bridges and has shown to be both profitable and provide good aerodynamic performance. The good aerodynamic performance of the trapezoidal box girders are manifested by low wind loading and high critical wind speed for flutter, however, another problem may arise which are vibrations induced by vortex shedding at low wind speeds.

This problem was first identified with the Great Belt East bridge in the spring of 1998 and with the Osteroy bridge in 1999 which both are suspension bridges. These vibrations were in both cases effectively reduced by mounting guide vanes at the joint between the bottom and the side plate of the box girders \cite{5}. This solution forces the flow to follow the lower contour of the box girder which reduces vortex shedding.

These bridges are both made with one single box girder i.e. single deck. Several bridges have also been constructed where the single deck has been split, creating two individual decks separated with a gap. These twin deck bridges are used as they show improved aerodynamic performance compared to the single deck bridges. The general understanding of this improvement of the aerodynamic performance, is that it is caused by increased aerodynamic damping generated by the introduction of the gap and the displacement of the decks. Some of these
twin deck bridges are Akashi Kaikyō Bridge, Xihoumen Bridge and Stonecutters Bridge.

The human desire of surpassing itself by building suspension bridges with longer and longer spans are a constant motivation for further research. This is also a necessity because when the spans are increased the sensitivity to the aerodynamics also increases. Furthermore solving possible issues after erection, as e.g. mounting guide vanes to reduce vortex induced vibrations, are not a profitable solution. This means that there is a high demand for identifying and addressing all possible issues before erection to avoid additional and unnecessary costs.

1.1 Problem statement

The influence of introducing a gap at the center of a simplified generic geometry resembling a bridge deck with respect to aeroelastic instability is studied. The aeroelastic instability or flutter presently in focus is more specifically classical flutter.
2 Governing equations

The aerodynamic forces on a body subjected to a free stream can be determined using different types of methods. The different types of methods available will not be either used or discussed as presently only one method is used. Therefore throughout this report the aerodynamic forces on a body subjected to a free stream is determined using a discrete vortex method implemented in the existing code DVMFLOW. DVMFLOW is a mesh free generic two-dimensional (2D) Navier-Stokes solver for bluff body flow. The method is Lagrangian and the dependent variable is the vorticity and the position of the vortices. The laminar diffusion is modeled with a random walk of the vortices. The theory and implementation of DVMFLOW will be explained in the following according to [8] and [6].

2.1 Mathematical formulation

2.1.1 Kinetic and kinematics relations

The kinetics of a 2D laminar flow with constant kinematic viscosity $\nu$ and density $\rho$ in a domain $D$ which is bounded by $\partial D = B$ is governed by the vorticity transport equation given by

$$\frac{\partial \omega}{\partial t} + (\vec{v} \cdot \nabla) \omega = \nu \nabla^2 \omega \tag{1}$$

where $\vec{v}(\vec{x}, t)$ is the velocity and $\vec{\omega}(\vec{x}, t) = \omega(\vec{x}, t) \vec{e}_z$ is the vorticity with $\vec{e}_z$ being the unit vector normal to $D$. The kinematic relation between the velocity and the vorticity can be expressed as an integral equation known as the Biot-Savart relation which is given by

$$\vec{v}(\vec{x}, t) = \vec{U} - \frac{1}{2\pi} \int_D \frac{\vec{\omega}_0 \times (\vec{x}_0 - \vec{x})}{|\vec{x}_0 - \vec{x}|^2} \, d\mathcal{D}_0$$

$$+ \frac{1}{2\pi} \int_B (\vec{n}_0 \cdot \vec{n}) (\vec{x}_0 - \vec{x}) - (\vec{n}_0 \times \vec{n}_0) \times (\vec{x}_0 - \vec{x}) \frac{d\mathcal{B}_0}{|\vec{x}_0 - \vec{x}|^2} \tag{2}$$

where $\vec{U}$ is the irrotational onset flow, $\vec{n}$ is the unit vector normal to the surface $B$ and $\vec{v}$ is the velocity at $B$. The boundary integral accounts for the vorticity not included in $D$. Therefore if $D$ contains only all non-zero vorticity the contribution from the surface integral vanishes.

2.1.2 Vorticity boundary conditions

The value of the vorticity at the solid boundary is found by using equation (2). The values of both the volume integral over $D$ and surface integral over $B$ are known, however the contribution from the surface velocity are not, which then leads to the following
2.2 Vortex method

\[ \int \int_{D_B} \frac{\vec{\omega}_0 (\vec{x}_0 - \vec{x}_B)}{|\vec{x}_0 - \vec{x}_B|^2} \, d\mathbb{D}_0 = \mathbb{I}(\vec{x}_B) + 2\pi \left[ \vec{U} - \vec{v}(\vec{x}_B) \right] \quad (3) \]

where \( D_B \) is a "thin" layer adjacent to \( B \) and the vector \( \mathbb{I}(\vec{x}_B) \) is the induced velocity from the vorticity both in the fluid and the solid, however except the vorticity at \( B \). The induced velocity is given by

\[ \mathbb{I}(\vec{x}_B) = \oint_B \frac{(\vec{n}_0 \cdot \vec{n}_0) (\vec{x}_0 - \vec{x}_B) - (\vec{n}_0 \times \vec{n}_0) \times (\vec{x}_0 - \vec{x}_B)}{|\vec{x}_0 - \vec{x}_B|^2} \, d\mathbb{B}_0 \]

\[ - \int \int_{D - D_B} \frac{\vec{\omega}_0 \times (\vec{x}_0 - \vec{x}_B)}{|\vec{x}_0 - \vec{x}_B|^2} \, d\mathbb{D}_0 \quad (4) \]

In the context of the vortex method it is convenient to introduce the surface vortex sheet \( \gamma \) defined by

\[ \frac{\partial \gamma}{\partial \vec{n}} = \vec{\omega} \quad (5) \]

Using the definition of the vortex sheet from equation (5) in equation (4) yields

\[ \int_B \frac{\gamma_0 \vec{e}_z \times (\vec{x}_0 - \vec{x}_B)}{|\vec{x}_0 - \vec{x}_B|^2} \, d\mathbb{B}_0 = \mathbb{I}(\vec{x}_B) + 2\pi \left[ \vec{U} - \vec{v}(\vec{x}_B) \right] \quad (6) \]

which can be recognized as a Fredholm equation of the first kind in the unknown \( \gamma_0 \). The solution in \( \gamma_0 \) is unique up to a constant i.e. an infinite number of solutions exist. The solution is made unique by imposing the Kelvin circulation theorem stating, that the rate of change of the total vorticity in both the solid \( S \) and the fluid \( F \) is zero.

\[ \frac{\partial}{\partial t} \int \int_{F+S} \omega \, d\mathbb{D} = 0 \quad (7) \]

Consequently if the total vorticity is zero at \( t = 0 \) it will remain zero for \( t > 0 \).

2.2 Vortex method

The Lagrangian solution to equations (1) and (2) include tracking of individual fluid elements \((\vec{x}_p, \Gamma_p)\) according to the ordinary differential equations given by

\[ \frac{d\vec{x}_p}{dt} = \vec{v}(\vec{x}_p, t) \quad (8) \]

\[ \frac{d\vec{\omega}}{dt} = \nu \nabla^2 \vec{\omega} \quad (9) \]
where the velocity $\vec{v}(\vec{x}_p, t)$ is approximated using equation (2) which gives

$$
\vec{v}(\vec{x}_p, t) = \vec{U} - \frac{1}{2\pi} \sum_{j=1}^{n_s} \oint_{\vec{B}_j} \frac{(\gamma_0 \vec{e}_z + \vec{v}_0 \times \vec{n}_0) \times (\vec{x}_0 - \vec{x}_p) - (\vec{v}_0 \cdot \vec{n}_0) (\vec{x}_0 - \vec{x}_p)}{|\vec{x}_0 - \vec{x}_p|^2} \, d\vec{B}_0
$$

$$
- \frac{1}{2\pi} \sum_{i=1}^{n_v} \Gamma_i \vec{e}_z \times (\vec{x}_i - \vec{x}_p) \frac{|\vec{x}_i - \vec{x}_p|^2}{|\vec{x}_0 - \vec{x}_p|^2}
$$

where $n_s$ is the number of solids and $n_v$ is the number of free vortices with a vortex strength of $\Gamma_i$. The present singularity in equation (10) is controlled by applying the Gaussian core function. The solids are approximated by polygons and the surface vortex sheet. The surface velocity is described by a linear variation when evaluating both equation (10) and (6). Equation (8) is solved numerically by standard ordinary differential equation methods and the diffusion equation (9) is approximated by random walk. The combined solution to equation (8) and (9) using Euler integration is

$$
\vec{x}_p^{k+1} = \vec{x}_p^k + \vec{v}(\vec{x}_p^k) \Delta t + \vec{\eta}_p
$$

where $\vec{x}_p^k$ is the position of the $p$th fluid element at the $k$th time step, $\Delta t$ is the time step and $\vec{\eta}_p$ is a random number with zero mean and variance $2\nu \Delta t$. The vortex sheets at the surface are transformed into vortex blobs and subsequently diffused into the flow by one-sided random walks. If vortices that are already in the flow enters the solids by a random walk they are removed from the calculations. The conservation of the total vorticity given by equation (7) is modified accordingly

$$
\sum_{i=1}^{m_j} \frac{\gamma_{ij}^k - \gamma_{ij}^{a,k-1}}{\Delta t} \Delta s_{ij} + 2A_j \frac{\Omega_j^k - \Omega_j^{k-1}}{\Delta t} = 0
$$

for each $j$th solid $m_j$ is the number of boundary elements, $\gamma_{ij}^k$ is the circulation released at the $k$th time step, $\gamma_{ij}^{a,k-1}$ is the annihilated circulation from the previous time step, $\Omega_j$ is the angular velocity and $A_j$ is the area of the solid. The kinematic relation given by equation (2) involves $O(n_s^2)$ operations per time step. This costly problem is overcome by using a fast adaptive multipole algorithm that reduced the number of operations to $O(n_s)$. Limiting the number of computational operations even more, is performed by joining vortices in the wake when they have traveled sufficiently far downstream giving a criteria of $|\vec{x}_p| > \epsilon_1$. Furthermore, two vortices are merged to satisfy the zeroth and first-order vorticity moment provided they satisfy the following criteria

$$
\left| \frac{\Gamma_i \Gamma_j}{\Gamma_i + \Gamma_j} (|\vec{x}_i - \vec{x}_j| < \epsilon_2
$$

15
2.2 Vortex method

2.2.1 Momentum balance

The aerodynamic forces are determined by a momentum balance or integration of the surface pressure. Considering a momentum balance gives the following total aerodynamic forces

\[ \vec{F} = -\rho \frac{d\vec{\alpha}}{dt} + \rho \sum_{j=1}^{n_s} \frac{d}{dt} \int_{S_j} \vec{v} d\mathbb{D} \]  

(14)

where \( \vec{\alpha} \) is the first-order moment of vorticity given by

\[ \vec{\alpha} = \int \int_{S} \vec{x} \times \vec{\omega} d\mathbb{D} \]  

(15)

For a general translating and rotating body described by the position of the center of rotation \( \vec{x}_{\text{rot}} \) and the rotation \( \theta \) equation (14) can, as given in [8], be rewritten to

\[ \vec{F} = -\rho \left( \frac{d\vec{\alpha}}{dt} \right)_{y} + \rho \frac{d}{dt} \sum_{j=1}^{n_s} \left[ \vec{v}_{\text{rot}} A_j + 2\Omega_j A_j \vec{e}_z \times \vec{x}_{\text{rot}} + 3\Omega_j \vec{e}_z \times \vec{M}_j \right] \]  

(16)

where \( \vec{v}_{\text{rot}} \) is the velocity of \( \vec{x}_{\text{rot}} \) and \( \vec{M}_j \) is the first-order moment of area of the \( j \)th solid given by

\[ \vec{M}_j = \int \int_{S_j} (\vec{x} - \vec{x}_{\text{rot}}) d\mathbb{D} \]  

(17)

The first-order moment of vorticity given by equation (15) is approximated using the following sum

\[ \vec{\alpha} \approx \sum_{i=1}^{n_v} \vec{x}_i \times \vec{e}_z \Gamma_i \]  

(18)

2.2.2 Surface pressure

For bluff bodies the main contributor to the forces are the pressure distribution. Imposing the no-slip boundary condition reduces the Navier-Stokes equations to the following

\[ \frac{1}{\rho} \frac{\partial p}{\partial \vec{s}} = -\nu \frac{\partial \vec{\omega}}{\partial \vec{m}} - a_s \]  

(19)
where $\vec{n}$ and $\vec{s}$ is the unit vector normal and tangential and $a_s$ is the tangential acceleration of the boundary. By neglecting the stream wise diffusion at the solid boundary equation (1) gives

$$\frac{\partial \omega}{\partial t} = \nu \frac{\partial^2 \omega}{\partial \vec{n}^2}$$  \hspace{1cm} (20)

which combined with equation (5) gives

$$\frac{\partial \gamma}{\partial t} = \nu \frac{\partial \omega}{\partial \vec{n}}$$  \hspace{1cm} (21)

Combining equations (21) and (19) gives

$$\frac{1}{\rho} \frac{\partial p}{\partial \vec{s}} = -\frac{\partial \gamma}{\partial t} - a_s$$  \hspace{1cm} (22)

The flux of the circulation at the $i$th boundary element of the $j$th solid is given by

$$(\frac{\partial \gamma}{\partial t})_{ij} \approx \gamma_{kj} - \gamma_{k-1} \Delta t$$ \hspace{1cm} (23)

Equation (22) is integrated using the discrete circulation flux given by equation (23) along the solid boundary to determine the pressure distribution

$$\frac{1}{\rho} \int_{B_j} \frac{\partial p}{\partial \vec{s}} dB = -\int_{B_j} \frac{\partial \gamma}{\partial t} dB - \int_{B_j} \vec{v}_s dB$$  \hspace{1cm} (24)

$$= -\int_{B_j} \frac{\partial \gamma}{\partial t} dB - \int \int_{S_j} \vec{\omega} \cdot \vec{e}_z d\mathcal{D}$$  \hspace{1cm} (25)

$$\approx -\sum_{i=1}^{m_p} \frac{\gamma_{ij} - \gamma_{k-1}}{\Delta t} \Delta s_{ij} - 2A_{ij} \frac{\Omega_j^{k} - \Omega_j^{k-1}}{\Delta t} = 0$$ \hspace{1cm} (26)

### 2.3 Aerodynamic derivatives

Several utility programs have been developed for DVMFLOW one being aero which extracts the aerodynamic derivatives $H_1^i...H_4^i$ and $A_1^i...A_4^i$. The derivatives can be determined using either the method of Fourier averaging or least squares where the Fourier averaging is the method used throughout this report. aero determines the aerodynamic derivatives, based on the forces acting on, and the position of a body undergoing known forced motion. The forced motions are described by
2.4 The two vortex model

The two vortex model described in [1] and [2] is a simple approach to prediction of the critical wind speed for flutter for a twin-deck bridge. Representing the upwind and downwind sections of the twin-deck, each are done with a bound vortex placed in the upwind 1/4 chord points (Γ₁ upwind, Γ₂ downwind), and the corresponding control points (CP₁ upwind, CP₂ downwind) placed in the 3/4 chord points, see Figure 1. Each section is extended along the x-axis with potential flow wake from the trailing edge (γ₁ upwind, γ₂ downwind). The upwind section wake is assumed to be terminated at the leading edge of the downwind section, whereas the downwind section wake is assumed to be infinite long. The twin-deck is assumed to pitch about the centroid located in the center of the gap between the two sections.

2.4 The two vortex model

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\[ h(t) = c_{h1} \sin (c_{h2}t) \]  (27)
\[ \theta(t) = c_{\theta1} \sin (c_{\theta2}t) \]  (28)

where \( h \) is vertical translation i.e. heave motion and \( \theta \) is rotation i.e. pitch motion. The aerodynamic lift and moment expressed by the aerodynamic derivatives as given in [8]

\[ L = \frac{1}{2} \rho U^2 c^2 \left( kH_1^* \frac{\dot{h}}{U} + kH_2^* \frac{\dot{\theta}}{U} + k^2 H_3^* \theta + k^2 H_4^* \frac{h}{c} \right) \]  (29)
\[ M = \frac{1}{2} \rho U^2 c^2 \left( kA_1^* \frac{\dot{h}}{U} + kA_2^* \frac{\dot{\theta}}{U} + k^2 A_3^* \theta + k^2 A_4^* \frac{h}{c} \right) \]  (30)

where \( L \) and \( M \) are the aerodynamic lift and moment respectively, and \( k \) is the reduced frequency given by

\[ k = \frac{2\pi f c}{U} \]  (31)

where \( f \) is the frequency and \texttt{aero} assumes the density \( \rho \), the onset flow \( U \) and the chord \( c \) to be unity. Equation (29) and (30) shows that \( H_1^* \), \( H_2^* \), \( A_1^* \) and \( A_2^* \) are related to heave motion, and \( H_3^* \), \( H_4^* \), \( A_3^* \) and \( A_4^* \) are related to pitch motion, which is why both types of motion are required to produce a full set of aerodynamic derivatives.

The aerodynamic derivatives \( H_1^*..H_4^* \) and \( A_1^*..A_4^* \) found with \texttt{aero} can be given an interpretation according to equation (29) and (30). The derivatives \( H_1^* \), \( H_2^* \), \( A_1^* \) and \( A_2^* \) are related to velocity, and \( H_3^* \), \( H_4^* \), \( A_3^* \) and \( A_4^* \) are related to position. Therefore they can be considered as damping and stiffness coefficients respectively.
2.4 The two vortex model

GOVERNING EQUATIONS

Figure 1: Schematic drawing of the two vortex model

In time dependent motion the bound vortices in the 1/4 chord points must be balanced by the trailing wakes due to Kelvin circulation theorem, given in equation (7). The strengths \( \Gamma_1 \) and \( \Gamma_2 \) of the bound vortices and the trailing wakes \( \gamma_1 \) and \( \gamma_2 \), and thus the magnitude of the cross-wind lift \( L \), are established from the Biot-Savart law by forcing the vertical velocity \( w = 0 \) at the 3/4 chord point of each element i.e. the solid wall boundary condition. Applying the constrain of \( w = 0 \) at the two control points \( CP_1 \) and \( CP_2 \) gives

\[
\begin{align*}
\frac{\Gamma_1}{2\pi(x_{CP_1} - x_{\Gamma_1})} &+ \frac{1}{2\pi} \int_{x_{CP_1}}^{\infty} \frac{\gamma_1(x, t)dx}{x_{CP_1} - x} + \frac{\Gamma_2}{2\pi(x_{CP_2} - x_{\Gamma_2})} \\
+ \frac{1}{2\pi} \int_{2db}^{\infty} \frac{\gamma_2(x - 2db, t)dx}{x_{CP_1} - x} + U\theta + \frac{b(d - \frac{1}{2})\hat{\theta}}{2} + \hat{h} &= 0 \quad (32)
\end{align*}
\]

\[
\begin{align*}
\frac{\Gamma_1}{2\pi(x_{CP_2} - x_{\Gamma_1})} &+ \frac{1}{2\pi} \int_{x_{CP_2}}^{\infty} \frac{\gamma_1(x, t)dx}{x_{CP_2} - x} + \frac{\Gamma_2}{2\pi(x_{CP_2} - x_{\Gamma_2})} \\
+ \frac{1}{2\pi} \int_{2db}^{\infty} \frac{\gamma_2(x - 2db, t)dx}{x_{CP_2} - x} + U\theta + \frac{b(d + \frac{1}{2})\hat{\theta}}{2} + \hat{h} &= 0 \quad (33)
\end{align*}
\]

Assuming that, the bound vorticity \( \Gamma \) is harmonic in time, the shed vorticity \( \gamma \) is harmonic in time and space and that the rate of change of \( \Gamma \) must be balanced by the strength per unit length of the trailing vortex wake moving downstream with \( U \), simplifies the equations (32) and (33) to

\[
\begin{align*}
\Gamma_1 \left(1 - ik \int_0^\infty \frac{e^{-ik\xi}d\xi}{-\frac{1}{2} - \xi} \right) - \Gamma_2 \left( \frac{1}{2d + \frac{1}{2}} - ik \int_0^\infty \frac{e^{-ik\xi}d\xi}{-(2d - \frac{1}{2}) - \xi} \right)
&= -2\pi bU \left( \theta + \frac{b(d - \frac{1}{2})\hat{\theta}}{2U} + \frac{\hat{h}}{U} \right) \quad (34)

\Gamma_1 \left( \frac{1}{2d + 1} - ik \int_0^\infty \frac{e^{-ik\xi}d\xi}{(2d - \frac{1}{2}) - \xi} \right) + \Gamma_2 \left( \frac{1}{2} - ik \int_0^\infty \frac{e^{-ik\xi}d\xi}{2 - \xi} \right)
&= -2\pi bU \left( \theta + \frac{b(d + \frac{1}{2})\hat{\theta}}{2U} + \frac{\hat{h}}{U} \right) \quad (35)
\end{align*}
\]

where \( i \) is the imaginary unit and \( \xi = x/b \) is a non-dimensional length. A
relation is established by equations (34) and (35) between the bound vorticity about each section and the effective angle of attack as seen by the sections. The desired cross-wind lift coefficient $C_{L}^{\Gamma}$ is related to the bound vorticity using the following

$$C_{L}^{\Gamma} = \frac{\Gamma}{Ub} = -2\pi\theta c(k) \quad (36)$$

where $c(k)$ is an equivalent of Theodorsen circulation function and equals $\Gamma$ for a unit effective angle of attack. Equations (34) and (35) defines two simultaneous equations for the $c(k)$ functions, which are denoted $c_1(k)$ and $c_2(k)$, to be solved for the upwind and downwind sections

$$\left[1 - ikIw \left(-\frac{1}{2}\right)\right] c_1(k) - \left[\frac{1}{2d + 1} - ikIw \left(2d - \frac{1}{2}\right)\right] c_2(k) = 1 \quad (37)$$

$$\left[\frac{1}{2d + 1} - ikIw \left(\frac{1}{2}\right)\right] c_1(k) + \left[1 - ikIw \left(-\frac{1}{2}\right)\right] c_2(k) = 1 \quad (38)$$

where $Iw(z)$ is a wake integral with the argument $z$ given by

$$Iw(z) = \int_{0}^{\infty} \frac{e^{-ik\xi}d\xi}{-z - \xi} \quad (39)$$

Evaluating the wake integral is not straightforward. However, by expressing $Iw(z)$ in terms of the exponential integral, and combining this with the cosine $Ci(z)$ and sine $Si(z)$ integral, gives an expression, which can be easily evaluated

$$Iw(z) = \cos(z)Ci(z) + \sin(z)Si(z) - \frac{\pi}{2} \sin(z) + i \left(\cos(z)Si(z) - \sin(z)Ci(z) - \frac{\pi}{2} \cos(z)\right) \quad (40)$$

where the cosine and sine integral can be found from

$$Ci(z) = 0.57722 + \ln(z) + \int_{0}^{z} \frac{\cos(t) - 1}{t} dt \quad (41)$$

$$Si(z) = \int_{0}^{z} \frac{\sin(t)}{t} dt \quad (42)$$

Once $c_1(k)$ and $c_2(k)$ are determined for a suitable range of $k$, the twin-deck lift and moment coefficients due to bound vorticity are obtained by summing the relevant coefficients for the individual sections. These lift and moment coefficients originates from the vortices located at the $1/4$ chord points. For practical sections the total lift and moment involves additional non-circulatory
components associated with the added mass of air and the finite section chord length. This gives the following lift \( C_L \) and moment \( C_M \) coefficients

\[
C_L = -\pi \left[ \left( \theta + i\frac{k}{b} h \right) c_s(k) + i\frac{k}{2} \theta \left( \frac{1}{2} c_s(k) - d c_d(k) \right) + \frac{k^2}{b} h - i k \theta \right] \quad (43)
\]

\[
C_M = \frac{\pi}{4} \left[ \left( \theta + i\frac{k}{b} h \right) \left( \frac{1}{2} c_s(k) + d c_d(k) \right) + i\frac{k}{2} \theta \left( \frac{1}{4} - d^2 \right) c_s(k) \\
+ k^2 \left( d^2 + \frac{1}{8} \right) - i \frac{k}{2} \theta \right] \quad (44)
\]

where the sum \( c_s(k) \) and the difference \( c_d(k) \) of the circulation functions are given by

\[
c_s(k) = c_1(k) + c_2(k) \quad (45)
\]

\[
c_d(k) = c_1(k) - c_2(k) \quad (46)
\]

The aerodynamic derivatives related to lift are given by

\[
H_1^* = \frac{\pi}{8} \Im \left( \frac{i c_s(k)}{k} \right) \quad (47)
\]

\[
H_2^* = \frac{\pi}{32} \Im \left( i \left( \frac{1}{2} c_s(k) - d c_d(k) \right) \left( \frac{c_s(k)}{k^2} \right) + c_s(k) \left( \frac{c_s(k)}{k^2} \right) \right) \quad (48)
\]

\[
H_3^* = \frac{\pi}{32} \Re \left( i \left( \frac{1}{2} c_s(k) - d c_d(k) \right) \left( \frac{c_s(k)}{k^2} \right) + c_s(k) \left( \frac{c_s(k)}{k^2} \right) \right) \quad (49)
\]

\[
H_4^* = -\frac{\pi}{8} \Re \left( \frac{i c_s(k)}{k} - 1 \right) \quad (50)
\]

where \( \Re(z) \) is the real and \( \Im(z) \) is the imaginary part of the argument \( z \). The aerodynamic derivatives related to moment are given by

\[
A_1^* = \frac{\pi}{32} \Im \left( i \left( \frac{1}{2} c_s(k) + d c_d(k) \right) \left( \frac{c_s(k)}{k} \right) \right) \quad (51)
\]

\[
A_2^* = \frac{\pi}{128} \Im \left( i \left( \frac{1}{2} - d^2 \right) c_s(k) - \frac{1}{2} \right) + \frac{i c_2(k) + d c_d(k)}{k^2} \quad (52)
\]

\[
A_3^* = \frac{\pi}{128} \Re \left( i \left( \frac{1}{2} - d^2 \right) c_s(k) - \frac{1}{2} \right) + \frac{i c_2(k) + d c_d(k)}{k^2} + d^2 + \frac{1}{8} \quad (53)
\]

\[
A_4^* = \frac{\pi}{32} \Re \left( i \left( \frac{1}{2} c_s(k) + d c_d(k) \right) \left( \frac{c_s(k)}{k} \right) \right) \quad (54)
\]
2.5 Critical wind speed for flutter

The oscillations during aeroelastic instability or flutter are most likely to involve non-linear aerodynamics. However, it has been shown and accepted that it can be treated as a linear process. Therefore the flutter analysis is based on standard consideration regarding stability of a linear elastic system i.e. small amplitudes and linear behavior of the structure. The instability of the structure occurs when more energy is extracted from the flow than is dissipated through structural damping. The critical flutter condition is defined as the limit between decaying and divergent oscillations of the structure. The meaning of decaying is decreasing amplitude and divergent is increasing amplitude toward infinity for the given oscillation. In the present work this condition is formulated as the critical wind speed for flutter i.e. the wind speed at which flutter will occur. The critical wind speed for flutter can be determined using different routines, presently the method of Theodorsen given in [7] and [3] is used. Note that [7] presents a routine based on six aerodynamic derivatives $H_1^*..H_3^*$ and $A_1^*..A_3^*$ where [3] present one based on eight aerodynamic derivatives $H_1^*..H_4^*$ and $A_1^*..A_4^*$. In this study the latter is presented and used.

The model for determination of flutter assumes the deck section to move as a rigid body with two degrees of freedom which are vertical translation $h$ and rotation $\theta$. The section has the mass $m$ per unit length, the moment of inertia $I$ per unit length, the vertical $\omega_h$ and rotational $\omega_\theta$ eigenfrequencies and corresponding vertical $\zeta_h$ and rotational $\zeta_\theta$ damping ratios relative to critical. By using the definitions given, the equations of motion for the section are given by

$$m \left[ \ddot{h} + e \dot{\theta} + 2 \zeta_h \omega_h \dot{h} + \omega_h^2 h \right] = L \quad (55)$$

$$I \left[ m \ddot{\theta} + \dot{\theta} + 2 \zeta_\theta \omega_\theta \dot{\theta} + \omega_\theta^2 \theta \right] = M \quad (56)$$

where the aerodynamic lift $L$ and moment $M$ are given by equation (29) and (30) respectively and $e$ is the distance the center of mass is displaced from the elastic center. Due to the symmetry of the geometry the center of mass is placed on the vertical centerline hence $e = 0$.

Observations show that $h$ and $\theta$ are harmonic in time with a common frequency $\omega$ at the critical wind speed for flutter $U_c$. Assuming that $h$ and $\theta$ have solutions proportional to $e^{i \omega t}$, where $i$ is the imaginary unit, and introducing the frequency ratio $X = \omega/\omega_h$ gives the following solutions to equations (55) and (56)
2.5 Critical wind speed for flutter

\[
\left[ -X^2 - 2i\zeta_h X + 1 - \frac{\rho B^2}{m} X^2 H_1^* - i\frac{\rho B^2}{m} X^2 H_1^* \right] \frac{h}{B} \\
+ \left[ -i\frac{\rho B^2}{m} X^2 H_2^* - \frac{\rho B^2}{m} X^2 H_3^* \right] \theta = 0
\] (57)

\[
\left[ -\frac{\rho B^4}{I} X^2 A_4 - i\frac{\rho B^4}{I} X^2 A_1 \right] \frac{h}{B} \\
+ \left[ -X^2 - 2i\zeta \frac{\omega g}{\omega_h} X + \left( \frac{\omega g}{\omega_h} \right)^2 - i\frac{\rho B^4}{I} X^2 A_2 - \frac{\rho B^4}{I} X^2 A_3 \right] \theta = 0
\] (58)

where \( B \) is the width of the bridge deck. Setting the determinant of equations (57) and (58) equal to zero results in a complex polynomial in \( X \) of fourth degree. Assuming that \( X \) is real at the flutter condition, gives the real part of the flutter determinant as

\[
X^4 \left[ 1 + \frac{\rho B^2}{m} H_4^* + \frac{\rho B^4}{I} A_3^* + \frac{\rho B^6}{mI} (A_7 H_4^* - A_5^* H_1^* - A_4^* H_3^* + A_1^* H_2^*) \right]
\]

\[
+ X^3 \left[ 2\zeta h \frac{\omega g}{\omega_h} m H_1^* + 2\zeta h \frac{\rho B^4}{I} A_2^* \right]
\]

\[
+ X^2 \left[ -\left( \frac{\omega g}{\omega_h} \right)^2 - 4\zeta h \zeta \frac{\omega g}{\omega_h} - 1 - \frac{\rho B^4}{I} A_3^* - \frac{\rho B^2}{m} \left( \frac{\omega g}{\omega_h} \right)^2 H_1^* \right]
\]

\[
+ \left( \frac{\omega g}{\omega_h} \right)^2 = 0
\] (59)

and the imaginary part of the flutter determinant as

\[
X^3 \left[ \frac{\rho B^2}{m} H_1^* + \frac{\rho B^4}{I} A_3^* + \frac{\rho B^6}{mI} (A_7 H_4^* + A_5^* H_1^* - A_4^* H_3^* - A_1^* H_2^*) \right]
\]

\[
+ X^2 \left[ -2\zeta h \frac{\omega g}{\omega_h} - 2\zeta h - 2\zeta g \frac{\rho B^2}{m} H_4^* - 2\zeta h \frac{\rho B^4}{I} A_3^* \right]
\]

\[
+ X \left[ -\frac{\rho B^2}{m} \left( \frac{\omega g}{\omega_h} \right)^2 H_1^* - \frac{\rho B^4}{I} A_2^* \right]
\]

\[
+ 2\zeta h \left( \frac{\omega g}{\omega_h} \right)^2 + 2\zeta h \zeta g \frac{\omega g}{\omega_h} = 0
\] (60)

The equations (59) and (60) are solved successively for different assumed reduced wind speed \( U_r = U/(f c) \) and the roots are plotted against \( U_r \). The flutter condition is then identified as the intersection point \((U_{r,c}, X_c)\) between these two curves. This intersection point defines the critical wind speed for flutter \( U_c \) as

\[
U_c = X_c U_{r,c} f_h B
\] (61)
3 Numerical validation

3.1 Flow conditions

The general flow of the simulations is laminar as no turbulence model or turbulent inflow is included. Furthermore the flow around a 2D geometry e.g. a bridge deck section, includes three-dimensional (3D) effects as well. Experience with simulations using these conditions has shown to provide good results that agrees well with experiments [4]. A laminar consideration is sufficient if the flow is subject to massive separation and if this separation is caused by sharp corners, as is the case in many bridge engineering applications.

The simulations are preformed non-dimensional where the Reynolds number is the important non-dimensional number characterizing the flow. The Reynolds number is defined as

\[ Re = \frac{Uc}{\nu} \]

where the \( U \) is the onset flow, \( c \) is the chord and \( \nu \) is the kinematic viscosity. In this context these parameters are chosen such that they provide the desired Reynolds number. Therefore is \( \nu \) not to be considered as reflecting any actual real life viscosity. Nevertheless for completeness are they given by \( U = 1 \text{ m/s}, \ c = 1 \text{ m} \) and \( \nu = 10^{-4} \text{ m}^2/\text{s} \) which gives a Reynolds number of \( Re = 10^4 \). This Reynolds number is sufficiently low for the results to be regarded as more than just a model and high enough for a boundary layer to develop. Throughout the simulations this Reynolds number will be used if nothing else is mentioned.

Additional (flow) parameters related to the vortices generated at each time step exist in DVMFLOW. The influence of these parameters on the simulations will not be investigated in the present work. The values of the present simulations have been used in several previous studies, including studies by COWI.

One of these parameters is the non-dimensional vortex blob radius \( \tilde{\sigma} \), where the value of 2 is a typical choice used for the vortices cores to overlap. Another is the maximum circulation \( \Gamma_{max} = 10^{-2} \text{ m}^2/\text{s} \) to be released by any one boundary element. The remaining parameters, related to the merging of two vortices, are based on satisfying two criteria. The first criterion is defined as, the zeroth and first-order vorticity moment given by equation (13) with a typical threshold value of 0.1, if satisfied the vortices are merged. The second criterion is defined as, when vortices are diffused a specified distance downstream of the geometry where the typical value is \( 3c \), if satisfied the vortices are merged.

Note that the determination of the critical wind speed for flutter requires the density \( \rho \) to be given. The density of the air is assumed to be \( 1.22 \text{ kg/m}^3 \).

3.2 Geometry

The shape and dimensions of a bridge deck is obviously of great importance for the flow and the resulting aerodynamic force, it will be subjected to. When the interest of the study is the influence of introducing a gap in the bridge deck the
geometry should be as simple as possible. This is to isolate the effect of the gap, or at least minimize the effect of the overall shape of the geometry. Even with this restriction of simplicity, the geometry should still show to be realistic.

An apparent choice could be a simple square geometry as used in [4]. However, this square geometry showed a rather poor performance in flutter due to the very sharp edges. Therefore, instead of the square geometry a slight variation of this, also used in [4], is chosen, where the square is fitted a leading and trailing nose, see Figure 2. This geometry and the results it yields, will be regarded as the reference state in the present work. The horizontal chord $c = 1$ will be used as the reference chord in DVMFLOW.

![Figure 2: Schematic drawing of the geometry used for the bridge deck where $c$ is the horizontal chord](image)

The introduction of a gap in the bridge deck is performed by dividing the initial geometry through the vertical centerline. This was done without changing any dimensions other than the chord of each of the two new created geometries. The two geometries are moved an equal distance in opposite directions generating the gap $d$, see Figure 3. When doing this the chord is not rescaled and its definition is kept in accordance with the initial block design.

![Figure 3: Schematic drawing of the geometry used for the bridge deck with a gap](image)

This geometry provides a good basis for investigating only the influence of introduction of the gap in the bridge deck. An obvious next step would be to study the influence of the shape of the geometry, together with the introduction
of the gap. This could provide information, whether the shape of the geometry or the introduction of the gap is the leading order effect.

Therefore a slightly modified geometry is studied where the lower edge toward the gap was removed. This is performed by making a straight cut from the center of the lower chord to the center of the face toward the gap, see Figure 4. Note that this cut is also introduced on the reference geometry without the gap. Even though the modified and initial reference geometry is expected to yield almost identical results, this is performed for the purpose of completeness.

![Figure 4: Schematic drawing of the modified geometry with and without a gap](image)

The geometric input to DVMFLOW is a panel file that consists of a number of panels defining the outer boundary of the geometry. A typical number of panels used to resolve the boundary of a geometry is 200-400 panels as used in [8], [6], [1] and [4].

Generating the panel files are performed with a MATLAB script written for this study, see Appendix D.1. The script can produce a panel file for both the initial and the modified geometry with and without a gap. The desired number of panels, which are given as input to the script, are then distributed along the outer boundary of the geometry. This is performed such that the panel length is as uniform as possible, however, due to different length of the sides on the geometry, this is not completely satisfied. When the gap is introduced, extra panels are added to the faces of the gap. This ensures the same distribution of panels along the surface both with and without the gap, which provide a valid ground for comparison. The panel length for the geometry is as mentioned not completely uniform, however, the deviation is expected to be very small. This is investigated for the reference geometry of varying resolution, with and without a gap, see Table 1. The table shows that the variation of the panel length for any resolution is insignificant.
3.3 Structural properties

Determination of the critical wind speed for flutter of a given geometry involves the structural dimensions and dynamic properties of this geometry. The geometry chosen for the study without a gap resembles the geometry of the Great Belt East bridge. Therefore it is considered to be reasonable to use the structural parameters and dynamic properties of the Great Belt East bridge for the flutter predictions. These are summarized in Table 2 and Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>( B )</td>
<td>31</td>
<td>m</td>
</tr>
<tr>
<td>Cable spacing</td>
<td>( a )</td>
<td>27</td>
<td>m</td>
</tr>
<tr>
<td>Cable mass</td>
<td>( m_c )</td>
<td>3.4 ( \cdot 10^3 )</td>
<td>kg/m</td>
</tr>
<tr>
<td>Mass (incl. cables)</td>
<td>( m )</td>
<td>22.74 ( \cdot 10^3 )</td>
<td>kg/m</td>
</tr>
<tr>
<td>Mass moment of inertia (incl. cables)</td>
<td>( I )</td>
<td>2.47 ( \cdot 10^6 )</td>
<td>kgm(^2)/m</td>
</tr>
</tbody>
</table>

Table 2: Structural parameters for Great Belt East bridge

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical eigenfrequency</td>
<td>( f_h )</td>
<td>0.100</td>
<td>Hz</td>
</tr>
<tr>
<td>Torsional eigenfrequency</td>
<td>( f_\theta )</td>
<td>0.278</td>
<td>Hz</td>
</tr>
<tr>
<td>Vertical damping relative to critical</td>
<td>( \zeta_h )</td>
<td>0.5</td>
<td>%</td>
</tr>
<tr>
<td>Torsional damping relative to critical</td>
<td>( \zeta_\theta )</td>
<td>0.5</td>
<td>%</td>
</tr>
</tbody>
</table>

Table 3: Dynamic properties for Great Belt East bridge

It is obvious that these structural parameters will give an error in the flutter predictions, when the gap is introduced. When two masses with a common center of gravity are separated, the mass moment of inertia will increase by the square of the distance they are moved. Therefore it is considered proper for reason of comparison, to develop a realistic model that takes the change of the width and the mass moment of inertia into account. The Great Belt bridge consists of a girder box and two cables, see Figure 5.
The girder box can be considered as a line distributed mass of width \( B + D \), where \( D \) is the size of the gap, and mass \( m - 2m_c \) and the cables can be considered as point masses with a distance of \((a + D)/2\) to the center of gravity. Using these assumptions a simple model for the mass moment of inertia \( I_{\text{cor}} \) and the width \( B_{\text{cor}} \) can be written as a function of the gap \( D \):

\[
I_{\text{cor}}(D) = \frac{1}{12} (B + D)^2 (m - 2m_c) + 2 \left( \frac{a + D}{2} \right)^2 m_c \tag{63}
\]

\[
B_{\text{cor}}(D) = B + D \tag{64}
\]

It is not necessary to correct the dynamic properties as the vertical eigenfrequency will remain constant and the change in the torsional eigenfrequency is included in the corrected mass moment of inertia.

### 3.4 Simulations

Two of the methods available for determination of the critical wind speed for flutter for a given geometry are given here. The first is to subject the geometry to forced pitch and heave motion, and then use the aerodynamic derivatives extracted from the force signal. The second is to elastically suspend the geometry and subject it to different incoming velocities and then observe whether the motion is stable or not. For reason of completeness and comparison both methods for determination of the critical wind speed for flutter will be used in this study.

#### 3.4.1 Forced motion

The forced motion approach is performed by forcing the geometry to undergo specified harmonic pitch and heave motion, around the center of gravity at different frequencies within a certain range. The simulation time for each frequency is adjusted, such that it corresponds to 10 consecutive periods for the given frequency. This is assumed sufficient for providing a good signal for the Fourier averaging, during the extraction of the aerodynamic derivatives. The
specified harmonic pitch and heave motion the geometry are subjected to, are given by equation (27) and (28) respectively.

\[
\begin{align*}
h(t) &= c_{h1} \sin (c_{h2} t) \\
\theta(t) &= c_{\theta1} \sin (c_{\theta2} t)
\end{align*}
\]

where the amplitudes \( c_{h1} \) and \( c_{\theta1} \) are given in radians and the frequencies \( c_{h2} \) and \( c_{\theta2} \) are given by \( 2\pi f \). The frequencies can, however, be expressed more conveniently by using the reduced wind speed \( U_r \) which results in the following

\[
\begin{align*}
c_{h2} &= 2\pi \frac{U}{U_r c} \quad (65) \\
c_{\theta2} &= 2\pi \frac{U}{U_r c} \quad (66)
\end{align*}
\]

The simulations for both types of motion are then conducted, for a number of evenly distributed reduced wind speeds within a chosen interval. This is performed for each case to establish the data series on which the flutter prediction is based. The time series of the force and position for each of the reduced wind speed is then post processed by the utility program \texttt{aero}. It could be chosen to discard some of the initial time series data to ensure that the data to be used represents a flow that can be considered developed. However, due to the forced motion, the flow can be considered developed after a few time steps, and therefore the entire time series are used throughout the present work. The post processing of each case in \texttt{aero} yields the variation of the aerodynamic derivatives as a function of the reduced wind speed. The critical wind speed for flutter is then determined with the flutter routine based on the variation of the aerodynamic derivatives, see Appendix D.2.

When considering the amplitude for the forced motion two things have to be taken into account. The signal quality and the flow separation. Signal quality is important as it provides a valid basis for extracting the aerodynamic derivatives. If the chosen amplitude is to small the force signal will be dominated or contaminated by numerical noise. However, is the chosen amplitude to large, further separation of the flow could be provoked. Therefore the choice is a compromise between ensuring a proper signal and preserving the physics of the flow. A typical choice of the amplitudes \( c_{h2} \) and \( c_{\theta2} \) for the simulations are in radians what corresponds to 3 degrees as used in [8], [1] and [4]. Note that when degrees are used to describe the amplitude of the heave motion, it is only for convenience. The heave motion is a vertical displacement, where the amplitude is given in radians that corresponds to the stated degrees.

The amplitude of 3 degrees for pitch and heave motion is chosen for the present study, however, a significant amount of preliminary simulations, consisting of around 500 individual signals, has shown it to be insufficient. The reason is, the constant amplitude used for the heave motion. With a constant amplitude for heave motion, the effective angles of attack (AOA) on the geometry vary according to the reduced wind speed used in the given simulation. More specifically
this means that when the reduced wind speed is increased i.e. equivalent to a decreased frequency, the effective AOA decreases. This is, however, not the case for pitch motion where the maximum instantaneous AOA actually corresponds to the given amplitude. The influence of a decreased instantaneous AOA is that the forces on the geometry decrease and thereby deteriorate the signal quality. In the preliminary simulations this issue was primarily observed at higher $U_r$ for both the pitch and the moment signal after the gap had been introduced, for examples see Appendix A.1.

Based on these observations regarding heave motion, the amplitude adjusted such that the maximum instantaneous AOA would correspond to the maximum instantaneous AOA during pitch motion. During heave motion the AOA is only present because of the motion of the geometry, and can be found as the angle between the wind relative to the geometry and the onset flow, as seen on Figure 6.

![Figure 6](image_url)

Figure 6: The wind relative $U_{rel}$ to the geometry and the onset flow $U$ defining the effective angle of attack (AOA) during downward heave motion with the velocity $U_{geo}$.

Adjusting the heave amplitude such that the maximum instantaneous AOA during heave matches the maximum during pitch, requires determination of the velocity of the geometry. The motion of the geometry during heave, is described by equation (27) and differentiated this yield the velocity of the geometry

$$U_{geo}(t) = c_h c_{h2} \cos(c_{h2}t)$$  \hspace{1cm} (67)

The maximum instantaneous AOA is observed when the geometry achieve its maximum velocity. The maximum velocity of the geometry is due to the harmonic nature simply given by the amplitude. By using the velocity amplitude from equation (67) and combining this with equation (65), this yield the maximum velocity of the geometry

$$U_{geo,max} = \frac{2\pi c_h U}{U_r c}$$  \hspace{1cm} (68)

By using trigonometry with the determined velocities on the geometry, see Figure 6, the maximum instantaneous AOA denoted by $\alpha_{max}$ is found as
\[ \alpha_{\text{max}} = \tan^{-1} \left( \frac{2\pi c_{h1}}{U_r} \right) \] (69)

where \( \alpha_{\text{max}} \) is given in radians. Now the expression for the heave amplitude \( c_{h1} \) in radians can be determined by rearranging equation (69) to

\[ c_{h1} = \frac{U_r}{2\pi} \tan (\alpha_{\text{max}}) \] (70)

To show the influence of the constant heave amplitude used in the preliminary simulations, the maximum instantaneous AOA is found for a couple of reduced wind speeds. The results are summarized in Table 4. The constant amplitude show a significant difference of a factor 5 between the lowest reduced wind speed \( U_r = 4 \) and the highest \( U_r = 20 \). It is evident, that using a constant heave amplitude provide very different conditions for extracting a force signal, within an interval of reduced wind speeds. For the remaining results presented, the heave amplitude used is set to what corresponds to 3 degrees in radians if nothing else is mentioned.

<table>
<thead>
<tr>
<th>( U_r )</th>
<th>( \alpha_{\text{max}} ) in degrees</th>
<th>Constant amplitude</th>
<th>Adjusted amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4.7018</td>
<td>3.0000</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1.5704</td>
<td>3.0000</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.9424</td>
<td>3.0000</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: The maximum instantaneous AOA in degrees at different reduced wind speed for both constant and adjusted heave amplitude

### 3.4.2 Elasticly suspended

The second method to determine the critical wind speed for flutter, is to suspend the geometry elasticly. This is performed by regarding the geometry as a dynamical system, that is free to move within 2 degrees of freedom, which are the vertical translation \( h \) and the rotation \( \theta \). Then by subjecting the geometry to different wind speeds, and observing the behavior of the two degrees of freedom, the point of instability can be determined. The instability occurs at a wind speed which is identified as the critical wind speed for flutter. The degrees of freedom in the dynamical system are governed by the following equations

\[ c_{h0} \ddot{h}(t) + c_{h1} \dot{h}(t) + c_{h2} h(t) + c_{h3} = L \] (71)

\[ c_{\theta 0} \ddot{\theta}(t) + c_{\theta 1} \dot{\theta}(t) + c_{\theta 2} \theta(t) + c_{\theta 3} = M \] (72)
Regarding the geometry as a dynamical system requires, that the structural parameters and the dynamic properties are represented during the simulations. This is obtained by assigning the coefficients $c_{h0}$, $c_{h1}$, $c_{h2}$, $c_{θ0}$, $c_{θ1}$, $c_{θ2}$, in equation (71) and (72), with the non-dimensional translatory inertia $m/(ρB^2)$ and rotary inertia $I/(ρB^4)$ as explained in [6]. Both the stiffness of the elastic supports, from where the geometry is suspended, and the damping are defined, such that they reflect the reduced wind speeds $U/(f_h B)$ and $U/(f_θ B)$ related to translation and rotation respectively. This is a convenient way to formulate the coefficients, as they make it possible to run simulations at a given wind speed $U_s$, i.e. by adjusting the frequency accordingly, and preserve the initial onset flow. The coefficients related to the translation are given by

$$c_{h0} = \frac{m}{ρB^2}$$
$$c_{h1} = 2ζ_h \left(2π \frac{f_h B}{U_s} \right) \frac{m}{ρB^2}$$
$$c_{h2} = \left(2π \frac{f_h B}{U_s} \right)^2 \frac{m}{ρB^2}$$

The coefficients related to rotation are given by

$$c_{θ0} = \frac{I}{ρB^4}$$
$$c_{θ1} = 2ζ_θ \left(2π \frac{f_θ B}{U_s} \right) \frac{I}{ρB^4}$$
$$c_{θ2} = \left(2π \frac{f_θ B}{U_s} \right)^2 \frac{I}{ρB^4}$$

Note that the last two coefficients $c_{h3}$ and $c_{θ3}$ are constants, which are only used for providing the geometry with an initial perturbation if necessary. These constants can be assumed to be zero if nothing else is mentioned.

### 3.5 Establishing accuracy

In this study there are four main factors that can influence the accuracy of the obtained result i.e. the critical wind speed for flutter. These factors are the integration scheme used for time stepping, the actual size of the time step, the number of panels used to resolve the geometry and the Reynolds number. As previously stated, the Reynolds number is $Re = 10^4$, therefore the influence of the Reynolds number will not be investigated further. The accuracy analysis will be based on the three factors remaining and their influence on the critical wind speed for flutter.

There are four integration schemes available in DVMFLOW where only the two of them will be used in this study. The first being the Euler scheme of order $O(1)$ and the second being a Runge-Kutta (RK) scheme of order $O(2)$. It could be
3.5 Establishing accuracy

argued that it is unnecessary to investigate the Euler scheme, as a scheme with higher order is available. However, the Euler scheme is of particular interest as this is the scheme used by COWI for their simulations with DVMFLOW. The reason why COWI uses Euler, is that it is less expensive in the sense that it requires less computational time, and that they have experienced good agreement between Euler and experimental results. Because of this, it is chosen to include Euler in the present study.

A typical choice of the non-dimensional time step $\Delta t = \frac{dt U}{c}$ used in DVMFLOW, is $\Delta t = 2.5 \cdot 10^{-2}$ as used in several studies e.g. in [8], [6], [1] and [4]. The reason for this choice is, however, not well documented but it will serve as the reference in this study. The influence of changing the time step size, will also be investigated for some cases to ensure the reliability of the obtained result.

As previously mentioned, the typical number of panels used to resolve a geometry are in the range of 200-400 panels. Again, the reason for this choice is not well documented and therefore it seems reasonable to investigate, if this actually is a sufficient resolution.

Evaluating the results, which in this case is the variation of the critical wind speed for flutter, becomes more thorough and valid, if there is a knowledge regarding the possible error associated with the result. As there is no obvious or clearly defined way to estimate the error of the obtained critical wind speed for flutter, a possible estimate is chosen for the purpose. The error estimation is performed using sub sampling on the force signal from the simulations, by evaluating the difference between using a part of or the entire available signal. It is noted that this error estimate only is related to the extracted aerodynamic derivatives, and not the geometric and time-wise resolution. The mathematical formulation of this error estimate, is the absolute difference between $U_c$ based on 10 and 5 consecutive periods, which leads to the following expression

$$U_{c,\text{error}} = |U_{c,10p} - U_{c,5p}|$$

(79)

The influence of the main factors on the reference geometry are investigated. The investigation consists of both Euler and RK time stepping, for a number of geometric resolutions varying from 100 to 800 panels, see Figure 7. Furthermore, for one of the resolutions an investigation regarding the size of the time step is performed. The general expectation to the results are, that convergence toward a constant value should be observed for increasing geometric resolution i.e. increasing number of panels.

In contrast to the expectation, the results obtained with Euler shows continuously decreasing values, with a total decrease of $20 \frac{m}{s}$, for increasing resolution i.e. not showing convergence. The error estimate shows values in the range of 0 to around $4 \frac{m}{s}$ for the different resolutions, which must be considered as relative small i.e. up to around 5% of the critical wind speed for flutter. Even when the results are considered together with the corresponding error estimate, convergence is still not achieved.

The results obtained with RK shows a different behavior. An increase of close to $30 \frac{m}{s}$ is observed between 100 and 300 panels, where for higher resolutions it somewhat follows the behavior of Euler and decreases around $15 \frac{m}{s}$, except
3.5 Establishing accuracy

for a slight increase between 700 and 800 panels. This change from decrease to increase shows that RK seems to have a converging tendency. The error estimate of RK display varying values in the range of $1 \text{m} \text{s}^{-1}$ to $13 \text{m} \text{s}^{-1}$. When considering the results together with the corresponding error, it becomes even more evident that RK shows convergence.

The accuracy of both Euler and RK seems to depend strongly on the resolution in the sense, that Euler performs best with a low resolution and RK performs best with a high resolution.

The investigation of the time step size is performed with a resolution of 400 panels for both Euler and RK. As stated, the reference time step is $\Delta t = 2.5 \cdot 10^{-2}$.

As observed, RK seems to provide reliable results and it is considered sufficient to only reduce the time step by half once $(\Delta t/2)$, when determining if the time integration is correct. The result obtained with the decreased time step show, compared to the the result with the reference time step, an insignificant difference. This shows that the reference time step is sufficiently small to ensure a stable solution for the time integration with RK.

In contrast, Euler seems to provide less reliable results, therefore the time step is reduced by half twice $(\Delta t/2$ and $\Delta t/4$) to determine if it converge toward RK. The results with the reduced time steps, clearly converge toward the corresponding RK value, with only a difference of around $2 \text{m} \text{s}^{-1}$. Furthermore, it is seen to be sufficient to only reduce the time step by half for achieving convergence with RK.

![Figure 7: The critical wind speed for flutter $U_c$ and an estimate of the associated error $U_{c,\text{error}}$, as function of number of panels used to resolve the reference geometry i.e. no gap](image)

It is expected that introducing the gap in the geometry will make the flow more complex. Based on this expectation, it is considered inadequate only to investigate the factors influence on the reference geometry. Therefore, it is also chosen to investigate the influence of the factors on the geometry with a gap of $D/T = 1$, where $T$ is the thickness of the geometry. This investigation also consists of both Euler and RK time stepping for resolutions varying from 120 to 692 panels, see Figure 9. Again, one of the resolutions is used for an investigation
regarding the size of the time step.

Again, the result with Euler shows no convergence, but a rather large deviation of magnitude $100 \, \text{m/s}$. The results obtained for resolutions of 348 to 692 panels are not considered reliable, based on the observed behavior of the aerodynamic derivatives related to the moment. These derivatives show significantly reduced values compared to the ones extracted from RK, see Figure 8. This is considered as the reason for the very poor results obtained with Euler. Compared to the previous Euler results, similar values for the error estimate are observed in the range 0 to around $5 \, \text{m/s}$, with one exception at 348 panels where the error is around $25 \, \text{m/s}$. Considering the results together with the corresponding error, is not considered necessary due to the unreliable results and the lack of convergence.

![Figure 8: The comparison of the extracted aerodynamic derivatives using Euler (dashed line) and RK (solid line), for the geometry resolved by 348 panels and with a gap $D/T = 1$](image)

Again, RK is considered to provide better results. These results show an increase of around $25 \, \text{m/s}$ between 120 and 234 panels, which is in good agreement with the previous results. For the resolutions of 234 to 578 panels a relative constant value is observed, and then a slight decrease of around $10 \, \text{m/s}$ between 578 and 692 panels. If the last decrease is neglected, RK can be considered as showing very good convergence. The error of RK shows varying values within the range of $3 \, \text{m/s}$ to $10 \, \text{m/s}$. The highest value of the error is seen for the highest resolution which can explain the decrease between 578 and 692. Taking this into account and considering the results together with the corresponding error, it becomes evident that RK shows convergence.

The investigation of the time step size is performed for a resolution of 464 panels. Based on the previous results, it is not considered necessary to reduce the time step for RK, but only to reduce the time step by half for Euler. Again, it is observed that by reducing the time step for Euler, the obtained result converge
3.5 Establishing accuracy NUMERICAL VALIDATION

toward RK. Furthermore, the Euler results are considered to be reliable by providing better aerodynamic derivatives.

In general RK is considered to be more versatile and provide stability when the flow characteristics are changed with a different geometry.

![Graph](image)

**Figure 9**: The critical wind speed for flutter $U_c$ and an estimate of the associated error $U_c,\text{error}$, as function of number of panels used to resolve the geometry with a gap of $D/T = 1$

From the previous discussion of the results, it is obvious that great caution should be taken when using Euler. Based on observation throughout this analysis, it is considered evident that Euler has a tendency to simplify the characteristics of the flow. The main indicators of this, have been the diffusion of the smaller vortex structures, and the occurrence of single degree of freedom flutter instead of two degree of freedom flutter.

The diffusion of vortex structures is best observed by comparing instantaneous flow plots obtained from both Euler and RK, at the same non-dimensional time $tU/c$, see Figure 10 and Figure 11. These flow plots clearly shows how a number of small vortices are present when using RK and not when using Euler. Especially it is noticeable and important along the boundary of the geometry, as the pressure determines the loads on the geometry. The wake structures are also more complex when using RK, which could have an influence on induced vibrations due to the vortex shedding from the geometry.

The occurrence of single degree of freedom flutter are based on the observations of the extracted aerodynamic derivatives more precisely $A^*_2$. The behavior of this derivative provide information regarding the type of flutter present. Single degree of freedom flutter is when the value of the derivative is negative for small $U_r$ and eventually becomes positive. Two degrees of freedom flutter are slightly different, as the derivative $A^*_2$ initially is negative and continuous to remain negative for increasing $U_r$. 

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Figure 10: Instantaneous ($tU/c = 25$) flow plots generated using Euler (a) and RK (b), showing the geometry with $D/T = 1$ undergoing pitch motion at the reduced wind speed $U_r = 20$

Figure 11: Instantaneous ($tU/c = 25$) flow plots generated using Euler (a) and RK (b), showing the geometry with $D/T = 1$ undergoing heave motion at the reduced wind speed $U_r = 20$
Based on this analysis, it is possible to make a qualified choice of the adequate geometric and time-wise resolution, to be used for the remaining simulations in this study. It could be argued that the highest geometric resolution should be chosen, as it must be regarded as providing the best accuracy. However, it is considered reasonable to use roughly 400 panels such that acceptable computational time can be obtained, but still ensuring a sufficient accuracy for the remaining analysis. More specifically the number of panels used, are 404 for the geometry with no gap and 464 for the geometry with a gap. The time stepping is chosen to be performed by RK as it provides the most stable and reliable results for both geometries. The time step of $\Delta t = 2.5 \cdot 10^{-2}$ is considered to be sufficient. These parameters are used in the following unless stated otherwise.

As stated, it is considered sufficient with a simulation time of 10 consecutive periods, to ensure reliable results when extracting the aerodynamic derivatives. To verify this assumption, the variation of the critical wind speed for flutter, as function of the number of included periods, is investigated. This is performed by extracting the aerodynamic derivatives from 1 to 10 periods of a signal consisting of 10 consecutive periods. This analysis is based on the reference geometry without a gap resolved by 404 panels and time stepped with RK. Figure 12 shows that the average $U_c$ has converged to a large extent from just 2 periods, after which it varies only by $2 \text{ m/s}$. The observed fluctuations have a random and decreasing nature, which shows that it is insufficient to use less than 10 consecutive periods.

![Figure 12: The critical wind speed for flutter $U_c$ as function of the number included simulation periods](image)

However, the required number of periods are conjectured to depend strongly on the quality of the signal, used for extracting the aerodynamic derivatives. If the quality of the signal is questionable, the periodicity may not be so easily identifiable which could possibly influence the result. However, this is not investigated further and the remaining simulations are done for 10 consecutive periods of time corresponding to the frequency.
3.6 Validation

Due to the great resemblance of the reference geometry to the Great Belt East bridge, it is possible to validate the results against one of the extensive studies of this bridge. Validating the results found for the geometry with a gap, however, is more difficult as few studies exist of twin-deck bridges. One of the studies of the Great Belt East bridge which is used in the present work, given in [4], provide prediction of the critical wind speed for flutter obtained both from DVMFLOW and from wind tunnel experiments, see Table 5.

Comparing these results, requires that the result obtained for the reference geometry is scaled because of a difference in the characteristic length. The characteristic length of the Great Belt East bridge geometry used in [4], is the chord from the leading to the trailing edge, whereas the present study it is defined as the horizontal chord, as shown in Figure 2. In effect of the two different definitions, the Great Belt East bridge chord is 1.32 times the present chord. It is possible to change the definitions and rescale time, forces, aerodynamic derivative, etc. However, it should be noted that this will effect the Reynolds number depending on the order of change. The aerodynamic derivatives obtained for the reference geometry is therefore scaled before the determination of $U_c$, see Table 5.

This result slightly deviates from the results obtained for the Great Belt East bridge both from DVMFLOW and experiments. However, as shown in Figure 7, a maximum error in the order of $3.5 \, \text{m/s}$ can be expected for $U_c$ with the used geometric resolution, time step size and time stepping scheme. Considering the estimated error, the critical wind speed for flutter for the reference geometry agrees well with the Great Belt East bridge.

<table>
<thead>
<tr>
<th>Reference geometry</th>
<th>Great Belt East bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td>DVMFLOW $U_c$[m/s]</td>
<td>77</td>
</tr>
<tr>
<td>DVMFLOW $U_c$[m/s]</td>
<td>73</td>
</tr>
<tr>
<td>Experimental results</td>
<td>70-74</td>
</tr>
</tbody>
</table>

Table 5: Comparison of the critical wind speed for flutter $U_c$ for the reference geometry and the Great Belt East bridge.
4 Results

4.1 Forced results

One method for determination of the critical wind speed for flutter $U_c$, is as previously explained based on the extracted aerodynamic derivatives. This method is the first to be used, for establishing the behavior of $U_c$, when a gap is introduced in the center of the reference geometry. Several gap sizes are chosen for this study in the range of $0 \leq D/T \leq 7.5$, where the main part is chosen within the range of 0 to 3. The simulations for each gap are chosen to cover the range of reduced wind speeds $4 \leq U_r \leq 20$, with increments of 2.

As stated, the resolution of the geometry is chosen to be 404 panels for the reference geometry and 464 panels for the geometry with a gap. For these resolutions $U_c$ have already been determined for the reference geometry and for the geometry with a gap of $D/T = 1$. However, due to convenience it is chosen to run these simulations again. Therefore, the simulations are performed for the entire range of chosen gaps and the results are shown in Figure 13. The results include both $U_c$ based on the constant inertia and the inertia corrected according to the gap.

The present simulation with the reference geometry yielded a critical wind speed for flutter $U_c = 55 \frac{m}{s}$, compared to the previously found $56 \frac{m}{s}$. This is a rather insignificant difference and less than the previously determined error estimate, of around $4 \frac{m}{s}$ for the given resolution. According to the previous validation, the new result is scaled and shows $U_c = 72 \frac{m}{s}$, which is in even better agreement with the results presented from the Great Belt East bridge.

With a gap of $D/T = 1$ the present simulation yielded $U_c = 81 \frac{m}{s}$, compared to the previously found $80 \frac{m}{s}$. Again, the difference is insignificant and less than the error estimate of around $3 \frac{m}{s}$ for the given resolution.

The variation of $U_c$, according to the gap $D/T$ with constant inertia, shows a significant increase initially toward the gap $D/T = 0.5$ of around 65%. After this, a slight drop is observed at $D/T = 1$, whereafter an increase is observed toward the maximum of around 85% at $D/T = 1.5$. Further a slight drop is seen that increases to around 84% at $D/T = 2.25$, whereafter it only decreases. The two largest gaps $D/T = 5 - 7.5$ shows rather small gains of around 22%, when compared to the small gaps. Note that the slight drops observed, seems to arise when the gap size $D$ corresponds 1, 2 and 3 times the thickness $T$ which will be discussed later in the report.

The initial rather significant increase found for a gap $D/T = 0.5$, is especially interesting. This is because it presents a potential for a reduction in the construction costs for a twin-deck bridge by using a small gap. With a small gap the amount of e.g. concrete used is significantly reduced. The gain in $U_c$ achieved by further increasing the gap, is rather modest compared to the large initial gain. For example, with a gap to $D/T = 1.5$ the increase in $U_c$ is 85% i.e. by making the gap 3 times larger the gain is only 20%.

A problem with the quality of the signal from pitch motion, primarily at high reduced wind speeds, was encountered for gaps of $D/T = 5$ and $D/T = 7.5$, for examples see Appendix A.2. These signals showed a tendency to include,
what is believed to be noise from the frequency of the vortex shedding from the upstream geometry. This problem is solved by increasing the amplitude of only the pitch motion to 5 degrees, for the reduced wind speeds in the range $16 \leq U_r \leq 20$.

The variation of $U_c$, with the corrected inertia according to the gap, shows in the range $0 \leq D/T < 1.5$ close to identical results compared to the constant inertia. For the remaining gaps mainly the magnitude of $U_c$ is affected, and an average increase of around 50% is observed compared to the constant inertia. The difference in inertia only become significant, by affecting the result, for a certain gap size. Therefore it can be considered as valid to use constant inertia up until $D/T = 1.5$. The results for gaps 5 and 7.5 are not shown, as they did not provide reliable results for unknown reasons. However, this is considered as of minor importance as the influence of correcting the inertia have been established.

The inertia correction is observed to mainly influence the magnitude in the range $1.5 \leq D/T \leq 3$, and not the actual behavior of $U_c$ i.e. show drops and peaks at same gaps, as found with constant inertia. Because of this, it is considered reasonable to use constant inertia in determination of $U_c$ for the remaining part of this study.

![Figure 13: The critical wind speed for flutter $U_c$ and the corresponding increase in % from the reference state i.e. $D/T = 0$, as function of the gap $D/T$ introduced in the center of the reference geometry](image)
Establishing the behavior of $U_c$, when a gap is introduced in the center of the corrected geometry, is performed with the same approach as to the reference geometry. These results should in comparison to the previous results, provide some indications regarding the influence and effect of a geometric change.

The gap sizes are chosen to be the same as the previously used in the range of $0 \leq D/T \leq 3$. The simulations with the corrected geometry are chosen to cover the range of reduced wind speeds $4 \leq U_r \leq 26$, with increments of 2. The range is chosen rather wide compared to the previously used, however, it is found necessary to ensure that the intersection, used for determination of $U_c$, in the flutter routine can be found. The resolution for the corrected geometry is slightly different from the reference geometry, by using 412 panels for the geometry with no gap and 442 panels for the geometry with a gap. The results obtained with the simulations performed, are for the entire range of the chosen gaps shown in Figure 14.

Again, a problem with the signal quality was encountered, more specifically the moment signal of both the heave and pitch simulations. This is also most prominent at the high reduced wind speeds, see Appendix A.3. The consequence of this problem, is that no result can be obtained for gaps $D/T$ of 1.75 and 1.875, and that a rather unexpected and significant drop in $U_c$ is observed around $D/T = 0.625 - 0.75$.

A possible solution to the problem, is to increase the amplitude of the heave and pitch motion during the simulations, for thereby increasing the moment signal. This possibility is pursued by increasing the amplitude from the initial 3 degrees to 5 degrees which showed no significant improvement. Therefore the amplitude is further increased to 8 degrees. This provides a better signal for the moment and the observed sudden drop in $U_c$ vanishes, however, it is not possible to determine $U_c$ for any gaps further than $D/T = 1$. Increasing the amplitude even more is considered as being trivial due to the previous discussed issues with large amplitudes. Another possible solution could be to reduce the time step which is not further pursued in this study.

The simulation with the corrected geometry with no gap, yielded a critical wind speed for flutter $U_c = 58 \text{ m/s}$ which shows good agreement compared to the previously found $U_c = 55 \text{ m/s}$ for the reference geometry. This is expected, as the geometries only differs by the slight triangle cut on the lower chord, which should be rather insignificant in the reference state.

The general behavior after the gap is introduced, if considering the combination of the results from the amplitude of 3 degree and 8 degree, is showing less variation compared to the previous result. The maximum increase of around 130% is in contrast to the reference geometry achieved at a gap of $D/T = 1$, where after for the remaining gaps the gain is seen to decrease slightly. This shows that the initial increase for the corrected geometry require a greater gap, however, the gain is twofold the gain obtained with the reference geometry. It is evident, even though the obtained signals are not completely convincing due to the discussed problems, that a geometric change can have a positive influence on $U_c$. 
4.1 Forced results

The two vortex model previously presented, is rather interesting as it can provide a very simple and costless alternative for determining the critical wind speed for flutter. Whether it is a good alternative to DVMFLOW depend highly on the accuracy of the determined $U_c$. Due to the simplicity of the model, one could expect, that it is unable to capture the complex flow around the geometry and thereby predict a correct $U_c$.

The simulations with the two vortex model are performed with a geometry resembling the reference geometry i.e. by having the same chord length. The gap sizes are chosen to be the same as the previously used in the range of $0 \leq D/T \leq 3$. The simulations for each gap are chosen to cover the range of reduced wind speeds $4 \leq U_r \leq 30$, with increments of 2. As previously, the range is chosen rather wide as it is found necessary to ensure the determination of $U_c$.

The two vortex model shows more than twice the $U_c$ as previously found for the reference geometry, see Figure 15. This is considered unrealistic, compared to the previous presented and validated result which should have been some what reflected by this model. The general behavior of $U_c$, after the gap is introduced, is a continuously almost exponential growth which must be considered unphysical. This model seems to be governed by one effect only, therefore not capturing the complexity of the flow surrounding the geometry. Based on these consider-
ations and the results, the two vortex model is considered to be too simple for providing reliable prediction of $U_c$.

During the present work, communication with COWI resulted in an effort to reproduce some results made by COWI. The results intended to be reproduced, are based on the Great Belt East bridge geometry with a gap introduced in the center of $D/T \approx 0.5$. The basis on which these simulations are performed, is slightly different in the sense of geometric resolution, integration scheme, time step, Reynolds number and whether the adjusted or constant heave amplitude is used. The results of this effort show relative good agreement with the previous presented results of $U_c = 92 \frac{m}{s}$ for $D/T = 0.5$, and can be seen as providing some verification, see Table 6.

These results show no sign of a problem with the constant heave amplitude as previously found with the reference geometry. Furthermore, the results show that a Reynolds number of $Re = 10^5$ only gives a small difference in $U_c$ compared to $Re = 10^4$. However, one should be careful to conclude anything based on this very limited analysis. Therefore these results will not be discussed or used further.


4.2 Elastic results

The second method for determination of the critical wind speed for flutter $U_c$, is as previously explained based on observing the motion of the geometry subjected to different incoming wind speeds. This method is used for comparison and validation of the previous obtained results based on the aerodynamic derivatives. The total simulation time for each wind speed is chosen to be $tU/c = 200$ which is considered a sufficient amount of time for observing the behavior of the geometry. Other similar studies have been performed with less simulation time e.g. $tU/c = 120$ as used in [6]. The choice of $tU/c = 200$ is used throughout the elastic simulations.

It is considered proper, to verify that the elastic simulations is correctly defined and reflect the given dynamic properties of the structure. DVMFLOW provide an option where the onset flow can be switched off during the entire simulation. This option is used to verify both the eigenfrequency and damping ratio given for the vertical and the rotational degree of freedom. This is performed by giving the geometry a slight perturbation in both the vertical and rotational direction, and then allowing it to move freely without the subjection to an onset flow. From this free motion the eigenfrequency and damping ratio can be identified. The eigenfrequency is identified from the period, which it oscillates with, and the damping ratio is identified from the amplitude decay of the oscillations for the given degree of freedom.

The verification of the elastic simulations are chosen to be performed with what is equivalent to an incoming wind speed of $U_s = 50 \frac{m}{s}$. Therefore the frequency of the oscillations from this simulation should reflect the frequency $f_iB/U_s$ it is assigned, see equations (74)-(75) and (77)-(78), where $i$ refers to $h$ and $\theta$. Based on the known non-dimensional time $tU/c$ of two successive peaks this frequency can be found. The frequency is given by

$$f = \frac{1}{n} \frac{1}{(tU/c)_n - (tU/c)_0}$$

(80)

where $(tU/c)_0$ is the non-dimensional time of the first peak and $(tU/c)_n$ is the
non-dimensional time of the peak $n$ periods away. Determining the damping ratio $\zeta$ of an under-damped system i.e. $0 \leq \zeta < 1$, in the time domain, can be performed by using the logarithmic decrement $\delta$ together with the displacement time history. Based on the known amplitude of two successive peaks, the logarithmic decrement can be found. The logarithmic decrement is given by

$$\delta = \frac{1}{n} \ln \frac{x_0}{x_n}$$

(81)

where $x_0$ is the largest amplitude of the two and $x_n$ is the amplitude of a peak $n$ periods away. The damping ratio based on the logarithmic decrement is given by

$$\zeta = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta}\right)^2}}$$

(82)

The motion of the vertical and the rotational degree of freedom, obtained from the simulation, show the expected behavior by oscillating at a constant frequency and having a decaying amplitude, see Figure 16. The amplitude peaks used for the verification are also shown. Based on these peaks the difference between the given and the estimated dynamic properties of the structure can be calculated, see Table 7. These values are insignificant which verifies that the given dynamic properties are in fact reflected during the elastic simulations. Note that the slight deviation observed, could be caused by the discretization of the signal not coinciding exactly with the peaks chosen.
4.2 Elastic results

The vertical $h$ and rotational $\theta$ motion during the simulation given an initial perturbation and not subjected to a onset flow is shown in Figure 16.

![Graph showing vertical and rotational motion](image)

Figure 16: The vertical $h$ and rotational $\theta$ motion during the simulation given an initial perturbation and not subjected to a onset flow.

<table>
<thead>
<tr>
<th>Degree of freedom $i$</th>
<th>The eigenfrequency $[s^{-1}]$</th>
<th>The damping-ratio $[-]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation $h$</td>
<td>$1.5504 \cdot 10^{-5}$</td>
<td>$3.1817 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>Rotation $\theta$</td>
<td>$5.3793 \cdot 10^{-6}$</td>
<td>$1.0635 \cdot 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 7: The absolute difference between the given and calculated dynamic properties of the structure

After having established that the simulation is correctly defined, such that it reflects the dynamic properties of the structure, a criterion for instability needs to be defined. As stated, the elastic simulations are performed with what is equivalent to a number of incoming wind speeds $U_s$ for each gap. The choice of the range of $U_s$, is based on the expectation that the elastic results will somewhat reflect the previous found forced results. For example it is chosen to run $D/T = 0.25$, which previously yielded a $U_c$ of around $75 \frac{m}{s}$, in the range $60 \frac{m}{s} \leq U_s \leq 100 \frac{m}{s}$ with increments of $5 \frac{m}{s}$, see Figure 17(a).

The vertical $h$ and rotational $\theta$ motion from each of the simulated wind speeds, will form the basis for identifying the point of instability and thereby the critical wind speed for flutter $U_c$. It is chosen to define the instability, as the point
between two of the simulated wind speeds, which displays a factor of at least 5 between their individual motion amplitudes. This definition is regardless of which degree of freedom that display this behavior first i.e. the first will define the point of instability. The argument for this choice is, that a sudden rather significant increase in the motion amplitude must represent an instability in the structure. The critical wind speed for flutter \( U_c \) is then chosen, based on the instability point, as the wind speed between the two wind speeds identified with the given definition. For example the previously mentioned gap \( D/T = 0.25 \) shows, at the chosen range of simulated wind speeds, a \( U_c \) that is identified to be \( 67.5 \) m/s, see Figure 17(b).

Note that even though a simulated wind speed is considered as stable from the given definition, the motion amplitudes can still show rather significant values which would be unacceptable for a real bridge. This fact is acknowledged, but neglected, as the stable wind speeds shows oscillations that neither decays or diverges, in context with the previously stated "The critical flutter condition is defined as the limit between decaying and divergent oscillations of the structure".
Figure 17: The vertical $h$ and rotational $\theta$ motion for all simulated wind speeds for a gap of $D/T = 0.25$ (a) and the two wind speeds at each side of the instability for the same gap (b).
As stated, there is potential of a significant reduction in construction costs of a bridge with twin-deck by using a small gap only. Therefore the elastic simulations are chosen to be performed in the range of $0 \leq D/T \leq 1$ with the previously used gaps. Furthermore, two larger gaps are chosen to validate the previously found results with the forced motion, more specifically the gaps of $D/T = 1.5$ and $D/T = 3$.

Note that the consequence of the definition of the instability and the wind speeds chosen for the simulations is, that the results for $U_c$ will have a maximum error of $\pm 2 \frac{m}{s}$.

The results obtained with both the elastic suspension and the forced motion simulations are shown Figure 18 for the range of $0 \leq D/T \leq 1$. The present simulation shows for the reference geometry a $U_c = 55 \frac{m}{s}$ which is identical to the previously found for the forced motion. This provides a further validation of the results obtained for the reference geometry.

The variation of $U_c$ according to the gap $D/T$ for the elastic simulations, shows slightly more conservative gain compared to the previous results. The initial increase toward $D/T = 0.5$ previously observed, is also seen with the present results, however, with a more modest slope displaying a gain of around 40% compared to the previous 65%. The maximum gain of around 40% is not associated with only one gap but the range $D/T = 0.5 - 0.75$. This is probably caused by the increments of $5 \frac{m}{s}$ used for the choice of $U_c$, as a smaller increment could provide a difference in $U_c$. The present results shows the same behavior as the previous, but not the same magnitude of the gain.

Ensuring that the results from the forced motion for increasing $D/T$ are valid, the two large gaps chosen will serve as validation of the results. The small gap chosen is as stated $D/T = 1.5$ and yields a $U_c$ of $87.5 \frac{m}{s}$, compared to $102 \frac{m}{s}$ found with the forced motion. Again, this shows that the elastic results are slightly more conservative than the forced motion by a reduction of $14.5 \frac{m}{s}$. The large gap $D/T = 3$, however, shows better agreement with a $U_c$ of $72.5 \frac{m}{s}$, compared to $74 \frac{m}{s}$ from the forced motion. Disregarding the difference in magnitude, a general agreement between the two methods of the behavior of $U_c$, is considered to be present.
Figure 18: The comparison of the critical wind speed for flutter $U_c$ and the corresponding increase in % from the reference state i.e. $D/T = 0$, found with the forced motion and elastically suspended analysis as function of gap in the range $0 \leq D/T \leq 1$. 

\[ \frac{U_c}{U_c(0)} - 1 \times 100 \% \]
5 Discussion

The observations which will be explained in the following, are considered as effects which possibly could be the reasons for the behavior of $U_c$. Only the effects seen on the initial reference geometry with and without a gap will be considered. However, these observations should still only be regarded as suggestions for causes, as only a rather limited analysis is performed and presented.

5.1 General

The general belief, among bridge engineering companies, regarding the introduction of a gap in the center of a bridge deck is, that the gain in $U_c$ is caused by aerodynamic damping. The aerodynamic damping increases as the two deck sections are displaced further apart from one another. Therefore the general belief is also that to ensure a significant gain, the gap should be rather large or at least a certain size.

This study shows, however, that the increase in $U_c$ is rather significant for small gaps where the aerodynamic damping is negligible. Therefore the gain in $U_c$ is considered to be caused by the pressure equalization between the upper and lower side of the deck and not by the aerodynamic damping.

In an effort to establish whether the pressure equalization is the leading effect, an investigation of the pressure distribution on the geometry is performed. More specifically the pressure distribution will be presented in the form of the pressure difference $\Delta p$ between the upper and the lower side of the geometry. It is chosen to present the pressure difference at positions of the geometry, according to the period, during one motion period. Actually, only half of the period is considered due to the symmetric geometry which should show identical results, but with opposite signs during downward and upward motion. The pressure difference for one motion period is established by using the average of 10 consecutive periods to ensure reliable data.

The simulations for this investigation are performed for a geometry with $D/T = 0$ and $D/T = 0.5$, both before and after the occurrence of instability i.e. at a $U_r$ corresponding to before and after the previously determined $U_c$. The reference geometry showed instability at what is equivalent to $U_r \approx 8$, therefore $U_c = 6$ and $U_r = 10$ are chosen for the simulations. Similarly showed the geometry with $D/T = 0.5$ instability at $U_r \approx 16$, therefore $U_r = 14$ and $U_r = 18$ are chosen for the simulations.

The pressure difference is for each simulation determined at four chosen positions during the motion period, more specifically the positions at $1/8$, $2/8$, $3/8$ and $4/8$ of a period, see Appendix B. The determined pressure differences for the geometries, before the occurrence of instability, at the position of $1/8$ period are shown in Figure 19(a). Note that the pressure difference is shown relative to the geometry such that the two geometries can be compared. The results from the heave motion show no significant difference between the two geometries. However, the results from pitch motion show a clear pressure equalization between the upper and lower side of the geometry with a gap. This is primarily affecting the upstream section, with a rather significant reduction of the pres-
sure difference, for the entire horizontal chord, compared to the geometry with no gap.

The pressure differences after the occurrence of instability, are shown in Figure 19(b). Again, the heave motion show no significant difference between the two geometries only some slightly increased values. The pressure equalization of the upstream section previously observed, is not as clear due to the present pressure difference at the trailing edge. Furthermore, the difference observed between the downstream section and the geometry with no gap is more prominent, by showing a high pressure difference at the stagnation point. These observations are considered to indicate, that the pressure equalization is the leading effect for the increase achieved in $U_c$. 
5.1 General Discussion

Figure 19: The pressure difference $\Delta p$ as function of the position relative to the geometry $x_{rel}$ at 1/8 period, for the geometry with $D/T = 0$ and $D/T = 0.5$ undergoing both heave and pitch motion, before (a) and after (b) occurrence of instability.
5.1 General

In an attempt to understand the difference between stability and instability, and identifying the initiation of instability, the net aerodynamic force $F_{aero}$ is considered. The net aerodynamic force, present due to the pressure difference experienced by the geometry, can be found by integrating the pressure difference along the geometry. The previously determined pressure differences are used for determining the net aerodynamic force, on the geometry with $D/T = 0.5$, as function of the position according to the period.

The results obtained for the upstream and downstream sections are shown in Figure 20(a) and Figure 20(b), respectively. The upstream section is observed to be nearly unaffected of the occurrence of instability, however, with one exception at 1/8 period where twice the magnitude in the net aerodynamic force for heave motion is observed. In contrast the downstream section shows that the instability clearly affects the net aerodynamic force. This agrees well with the previous observed behavior of the pressure difference after the occurrence of instability. This observation shows no evidence whether the downstream section, is either causing the instability or only is affected by the instability. In general the difference between stability and instability is clearly observed, however, it is not possible to identify a cause of instability.

![Figure 20: The net aerodynamic force $F_{aero}$ on the upstream (a) and the downstream (b) section as function of the position according to the period, for the geometry with $D/T = 0.5$ undergoing both heave and pitch motion, at a reduced wind speed $U_r = 14$ before and $U_r = 18$ after instability](image)
Another approach to identifying the initiation of instability is to visually investigate the flow experienced by the geometry. The geometry with $D/T = 0.5$ is also chosen for this investigation. The flow plots are generated, each at a step of $1/8$ of a period, for one motion period, see Appendix C. From these generated flow plots, it is not possible to observe or identify any specific behavior of the flow regarding stability.

5.2 Specific

In an attempt to understand the more specific behavior i.e. the peaks and drops of the $U_c$ for the reference geometry with a gap, the force amplitudes which the geometry are subjected to, are considered. The force amplitudes obtained during the heave simulations at a reduced wind speed of $U_r = 20$, will be used. The reason for this choice is, that the geometry is as stationary as possible, which is regarded as the optimal conditions for investigating the forces. The force amplitudes presented for each gap are normalized using the force amplitude of the reference geometry, see Figure 21.

The general behavior of the force amplitude is seen to show relative constant values up until $D/T = 2.5$, whereafter a sudden significant increase for the moment and a more modest decrease for the lift and drag is observed. In the range $0 \leq D/T \leq 2.5$ the variation is of much smaller magnitude, however, a variation is present which will be investigated. The first drop is the moment and occurs in the vicinity of where $U_c$ shows the large initial increase. The second drop is more wide and seem to capture both of the next two peaks for $U_c$, but not as convincing as the first drop. These observations seems to indicate that when $U_c$ increases, the moment amplitude decreases. Whether one is caused by the other is difficult to say. The lift and drag seems not to provide any useful information.

As previously stated, the slight drops observed for $U_c$ seems to arise when the gap size $D$ corresponds 1, 2 and 3 times the thickness $T$. This behavior is believed to be related to the size of the vortex structures shed from the upstream...
5.2 Specific discussion

geometry. The vortex size will reflect some length scale present in the geometry as e.g. the thickness. If this is the case, a vortex could possibly be trapped in the gap and thereby block or reduce the pressure equalization between the upper and lower side. Investigating this further is performed by examining flow plots of the geometry to see whether such behavior can be identified. A random flow plot is chosen which shows the geometry with a gap of $D/T = 1$ undergoing heave motion, at a reduced wind speed of $U_r = 20$, see Figure 22.

These flow plots show the expected behavior, where a vortex with a size that seems to reflect the thickness, is located in the middle of the gap. It seems reasonable to assume that, this vortex will reduce the flow through the gap when observing how the entire gap is filled. Identifying similar vortex structures for the gaps $D/T = 2 - 3$ are not that straightforward, therefore no observations are presented regarding these gaps.

Figure 22: Instantaneous flow plots showing the geometry with a gap of $D/T = 1$ undergoing heave motion, at a reduced wind speed of $U_r = 20$ (a) and closeup of the gap (b)
6 Conclusion

The work presented in this report, is a study of the influence of introducing a gap in a generic geometry according to aeroelastic instability i.e. more specifically classical flutter, presented in the form of the critical wind speed for flutter $U_c$.

The aerodynamic forces on a body subjected to a free stream are determined using the discrete vortex method implemented in the existing code DVMFLOW. The utility program aero is used for extracting the aerodynamic derivatives. A simple approach to predicting $U_c$ for a twin-deck bridge is presented and used. The theory for determination of $U_c$ based on the aerodynamic derivatives, is presented and implemented in a flutter routine.

A slight variation of a square geometry is used where a square is fitted with a leading and trailing nose. The introduction of the gap is performed by dividing the geometry through the vertical centerline. A slightly modified geometry is also chosen, where the lower corner toward the gap is removed on both sections, to study the influence of a geometric change. Generating the panel-files according to the geometry used in DVMFLOW, are performed with the Matlab script developed for the purpose.

Determining $U_c$ of a given geometry requires knowledge of the structural dimensions and dynamic properties of this geometry. Because of the resemblance between the generic geometry and the Great Belt East bridge, it is considered reasonable to use the structural parameters and dynamic properties of the Great Belt East bridge. Furthermore, a more realistic model for the variation of the mass moment of inertia according to the size of the gap, is developed.

Two methods are used for determining $U_c$ for a given geometry, mainly to provide good basis for comparison and validation. The first method subjects the geometry to forced pitch and heave motion in an onset flow. The aerodynamic derivatives are extracted from the force signal and used in the flutter routine to determine $U_c$. The second method elastically suspends the geometry and subject it to different incoming wind speeds, where the lowest wind speed showing divergent motion amplitudes of the geometry define $U_c$.

Preliminary simulations showed it to be insufficient to use a constant amplitude for heave motion, as the effective angles of attack on the geometry vary according to the reduced wind speed used. More specifically this means that when the reduced wind speed is increased, the effective angle of attack decreases. The influence of the decrease in angle of attack is that the signal quality deteriorates. Therefore it has been chosen to adjust the amplitude of heave motion such that the maximum instantaneous angle of attack, would correspond to the maximum instantaneous angle of attack during pitch motion.

An estimate of the associated error with the determination of $U_c$ is developed. This estimate is used in a thorough analysis to establish the proper choice for obtaining accuracy of the results. The analysis showed that a geometric resolution of around 400 panels, time stepping with RK and a time step of $2.5 \times 10^{-2}$ provided sufficient accuracy. Several problems with Euler are identified, indicating that great caution should be taken when using Euler. It is shown that 10 periods are sufficient simulation time to ensure reliable results.
The results obtained for the reference geometry are validated against existing results for the Great Belt East bridge showing good agreement. The assumption of the resemblance between reference geometry and Great Belt East bridge, is considered reasonable based on the validation of the results.

The variation of $U_c$ for the reference geometry, according to $D/T$ with constant inertia, shows a significant initial increase toward $D/T = 0.5$ of around 65%. This initial increase is especially interesting as it presents a potential for reducing the construction costs. Small variations are observed for the remaining gaps reaching the maximum increase of around 85% at $D/T = 1.5$. The largest gaps shows small gains of around 22%. Generally, the gain in $U_c$ achieved by increasing the gap further than $D/T = 0.5$, is rather modest compared to the initial gain.

The pitch signal at high reduced wind speeds for $D/T = 5 - 7.5$ showed a tendency to include, what is believed to be noise from the frequency of the vortex shedding from the upstream geometry. This is solved by increasing the pitch motion amplitude to 5 degrees in the range $16 \leq U_r \leq 20$.

The developed model for the inertia correction is observed to mainly influence the magnitude and not the actual behavior of $U_c$. Based on this observation, it is considered reasonable to use constant inertia in determination of $U_c$ for the remaining parts of the study.

The corrected geometry shows, if considering the combination of the results from the amplitude of 3 degree and 8 degree, less variation compared to the reference geometry. The maximum increase of around 130% is achieved at $D/T = 1$, where for the remaining gaps the gain slightly decreases. The initial increase for the corrected geometry requires a greater gap than the reference geometry, however, the gain is twofold. It is evident, even though the obtained signals are not completely convincing, that a geometric change can have a positive influence on $U_c$.

The moment signal of both the heave and pitch simulations showed poor signal quality, most prominent at the high reduced wind speeds. The consequence of this is that no results are obtained for $D/T = 1.75 - 1.875$ and an unexpected and significant drop in $U_c$ around $D/T = 0.625 - 0.75$. This is solved by increasing the amplitude from the initial 3 degrees to 8 degrees which provides a better signal for the moment and the sudden drop in $U_c$ vanishes. However, with this solution it is not possible to determine $U_c$ for $D/T > 1$.

The two vortex model shows more than twice the $U_c$ found previous for the reference geometry, and the general behavior is a continuously almost exponential growth, which must be considered unphysical. This model seems to be governed by one effect only, therefore not capturing the complexity of the flow. The model is considered to be too simple for providing reliable predictions of $U_c$.

The elastic simulations in the range $0 \leq D/T \leq 1$ shows slightly more conservative gain compared to the previous results. The initial increase toward $D/T = 0.5$ displays a gain of around 40% compared to the previous 65%. The maximum gain of around 40% is found for the range $D/T = 0.5 - 0.75$. Two larger gaps are also chosen for validation of the previously found results. A general agreement, of the behavior of $U_c$ between the elastic and forced methods, is considered to be present.
The pressure distribution on the geometry is studied to establish, whether the pressure equalization is the leading effect. The upstream section for the geometry with $D/T = 0.5$ show a clear pressure equalization and significant reduction in pressure difference, compared to the reference geometry. Therefore it is considered that the pressure equalization is the leading effect.

The difference between stability and instability is investigated using the net aerodynamic force before and after instability. The upstream section is nearly unaffected in contrast to the downstream section that clearly is affected by the instability. Furthermore, flow plots are also examined for a geometry with $D/T = 0.5$ to identify the difference, however, no specific behavior of the flow regarding stability could be observed.

The more specific behavior i.e. the peaks and drops of the $U_c$ for the reference geometry with a gap, is investigated. The force amplitudes obtained during heave motion at $U_r = 20$, are considered, and seems to indicate that an increase in $U_c$, decreases the moment amplitude. The slight drops in $U_c$ seems to arise at a certain gap size, which could be caused by vortex structures with a similar size. Flow plots show that a vortex with a size reflecting the thickness, is located in the middle of the gap and seems to reduce flow through the gap i.e. reduced pressure equalization.
7 Future work

Motivated by the favorable results found, it is the intention to try getting the results publish at e.g. Journal of Fluids and Structures. It is also under consideration whether the natural extension of the present work, should define a new possible Ph.d. project. Based on the present work, several questions have emerged which could provide the basis for further research. Some of the questions to be answered could be

Can the found results be verified using experiments e.g. in the form of wind tunnel tests?
Is these favorable results only found because of a rather simplified geometry?
Can a general behavior of the critical wind speed for flutter for any geometry with a gap be established?
How are the results affected by changing the Reynolds number?
What is the influence of further geometric changes?
References


Appendix
A Force signals

A.1 Preliminary test

Examples of the force signal and the associated fit, from before and after the heave amplitude adjustment from constant to adjusted for $4 \leq U_r \leq 20$. 
A.1 Preliminary test

Figure 23: The force signal and associated fit from the geometry with $D/T = 0.5$, undergoing heave motion at $U_r = 20$ before (a) and after (b) adjustment of the heave amplitude.
Figure 24: The force signal and associated fit from the geometry with $D/T = 1.5$, undergoing heave motion at $U_r = 20$ before (a) and after (b) adjustment of the heave amplitude
A.2 Reference geometry

Examples of the force signal and the associated fit, from before and after increasing the pitch amplitude from 3 degrees to 5 degrees for $16 \leq U_r \leq 20$. 
Figure 25: The force signal and associated fit from the geometry with $D/T = 5$, undergoing pitch motion at $U_r = 16$ before (a) and after (b) the increase of the pitch amplitude.
Figure 26: The force signal and associated fit from the geometry with $D/T = 5$, undergoing pitch motion at $U_r = 20$ before (a) and after (b) the increase of the pitch amplitude.
A.3 Corrected geometry

Examples of the force signal and the associated fit, from before and after increasing both the pitch and the heave amplitude from 3 degrees to 8 degrees for $4 \leq U_r \leq 26$. 
Figure 27: The force signal and associated fit from the geometry with $D/T = 1$, undergoing heave motion at $U_r = 20$ before (a) and after (b) adjustment of the amplitude.
Figure 28: The force signal and associated fit from the geometry with $D/T = 1.75$, undergoing heave motion at $U_r = 20$ before (a) and after (b) adjustment of the amplitude.
B Pressure distribution

Examples of the pressure difference experienced by the geometry with $D/T = 0$ and $D/T = 0.5$, undergoing both types of motion, before and after the occurrence of instability.
Figure 29: The pressure difference $\Delta p$ experienced by the geometry with $D/T = 0$, undergoing both types of motion at $U_r = 6$ (a) and at $U_r = 10$ (b), before and after the occurrence of instability respectively.
Figure 30: The pressure difference $\Delta p$ experienced by the geometry with $D/T = 0.5$, undergoing both types of motion at $U_r = 14$ (a) and at $U_r = 18$ (b), before and after the occurrence of instability respectively.
C  Flow visualization

One period of simulation, showed by the instantaneous flow plots with a step of one eight of a period. The geometry with $D/T = 0.5$ is undergoing pitch motion, at a reduced wind speed corresponding to before $U_r = 14$ and after $U_r = 18$ instability occurs.
Figure 31: Instantaneous flow plots generated using RK showing the geometry with $D/T = 0.5$ undergoing pitch motion at a reduced wind speed of $U_r = 14$
(a) ($tU/c=1.75$), (b) ($tU/c=3.5$), (c) ($tU/c=5.25$) and (d) ($tU/c=7$)
Figure 32: Instantaneous flow plots generated using RK showing the geometry with $D/T = 0.5$ undergoing pitch motion at a reduced wind speed of $U_r = 14$ (a) $(tU/c=8.75)$, (b) $(tU/c=10.5)$, (c) $(tU/c=12.25)$ and (d) $(tU/c=14)$
Figure 33: Instantaneous flow plots generated using RK showing the geometry with $D/T = 0.5$ undergoing pitch motion at a reduced wind speed of $U_r = 18$ (a) $tU/c=2.25$, (b) $tU/c=4.5$, (c) $tU/c=6.75$ and (d) $tU/c=9$
Figure 34: Instantaneous flow plots generated using RK showing the geometry with $D/T = 0.5$ undergoing pitch motion at a reduced wind speed of $U_r = 18$
(a) ($tU/c=11.25$), (b) ($tU/c=13.5$), (c) ($tU/c=15.75$) and (d) ($tU/c=18$)
D Developed tools

The scripts developed for this work is performed with Matlab R2006a on Windows XP.

Several different small scripts was produced for extracting force amplitudes, pressure distributions etc. These scripts are made specifically for the present choice of geometric resolution and panel orientation, which means that they need to be rewritten if another geometry is adopted. Based on this fact, these scripts are not shown here.

D.1 Panel file generator

Description
A script for generating the reference geometry or the corrected geometry, with or without a gap and writing it to the panel-file used by DVMFLOW

Input
The gap size \( g \)
The type of geometry \( geo \) which is 1 for reference geometry and 2 for corrected geometry
The number of wanted panels \( p \)

Output
A panel-file *.p containing the information regarding the geometry

```
1 %#################################################
2 %
3 % Script for generating panel file for bridge
4 % deck of square cross section with leading and
5 % trailing nose(Larsen,1998c), furthermore can
6 % a defined gap be introduced in the center, and
7 % the edge between the lower chord and the gap
8 % can be modified.
9 %
10 % Morten Plum DTU
11 % s042698 20-04-2010
12 %
13 %#################################################
14 clc
15 clear all
16
17 % Length of horizontal chord
18 B = 1;
19 % Height of bridge deck
20 h = 0.2*B;
21 % Gap between decks
22 g = 0.1;
23
24 % Geometry (1 squarenose, 2 squarenose with modifications)
25 geo = 1;
```
% Wanted number of panels (can deviate slightly with no gap
% and significantly with gap)

\[ p = 100; \]

% Leading and trailing edge of the bridge
\[ x_l = -0.16 \times B - \frac{g}{2}; \]
\[ x_t = B \times 1.16 + \frac{g}{2}; \]

% Length of nose sides and cut for modified geometry
\[ \text{Ledge} = \sqrt{(0.16 \times B)^2 + \left(\frac{h}{2}\right)^2}; \]
\[ \text{Lcut} = \sqrt{(B/4)^2 + \left(\frac{h}{2}\right)^2}; \]

% Total length of the combined sides and first estimated panel length
\[ L_{\text{total}} = 2 \times B + 4 \times \text{Ledge}; \]
\[ L_p = L_{\text{total}} / p; \]

% Choice according to gap
if \( g = 0 \)
  % Choosing number of panels for the different sides
  \[ \text{pe} = \text{ceil}(\text{Ledge} / L_p); \]
  \[ \text{pc} = \text{ceil}(B / L_p); \]
  \[ \text{pcl} = \text{ceil}(B/4 / L_p); \]
  \[ \text{pcut} = \text{ceil}(\text{Lcut} / L_p); \]
  
  % Creating general positions \((x,y)\) for the panels
  \[ X_{el} = \text{linspace}(-0.16 \times B, 0, \text{pe}+1); \]
  \[ X_{et} = \text{linspace}(1.16 \times B, B, \text{pe}+1); \]
  \[ X_c = \text{linspace}(0, B, \text{pc}+1); \]
  \[ Y_e = \text{linspace}(0, \frac{h}{2}, \text{pe}+1); \]
  \[ Y_c = \text{linspace}(\frac{h}{2}, \frac{h}{2}, \text{pc}+1); \]

% Choice according to geometry type
if \( \text{geo} = 1 \)
  % Total number of panels
  \[ \text{ptotal} = 4 \times \text{pe} + 2 \times \text{pc}; \]
  
  % Plotting the geometry
  \[ \text{plot}(X_c, Y_c, 'r.', X_{el}, -Y_e, 'b.', ... \]
  \[ , X_{el}, Y_e, 'm.', X_{et}, -Y_e, 'm.' ... \]
  \[ , X_{et}, Y_e, 'k.', X_{et}, -Y_e, 'k.'); \]
  \[ \text{axis equal} \]

% Opening panel-file
\[ \text{fid} = \text{fopen}('\text{squarenose}\_\text{single.p}', 'w'); \]
elseif \( \text{geo} = 2 \)
  % Total number of panels
  \[ \text{ptotal} = 4 \times \text{pe} + \text{pc} + 2 \times \text{pcl} + 2 \times \text{pcut}; \]
  
  % Creating additional positions \((x,y)\) for the panels for the cut
  \[ X_{c1} = \text{linspace}(0, B/4, \text{pcl}+1); \]
  \[ X_{c2} = \text{linspace}(3 \times B/4, B, \text{pcl}+1); \]
  \[ X_{\text{cut1}} = \text{linspace}(B/4, B/2, \text{pcut}+1); \]
  \[ X_{\text{cut2}} = \text{linspace}(B/2, 3 \times B/4, \text{pcut}+1); \]
  \[ Y_{\text{cl}} = \text{linspace}(-h/2, -h/2, \text{pcut}+1); \]
  \[ Y_{\text{cut1}} = \text{linspace}(-h/2, -h/2, \text{pcut}+1); \]
  \[ Y_{\text{cut2}} = \text{linspace}(0, -h/2, \text{pcut}+1); \]

% Plotting the geometry
\[ \text{plot}(X_c, Y_c, 'r.', X_{c1}, Y_{\text{cl}}, 'b.', X_{c1}, Y_{\text{cut1}}, 'g.' ... \]
  \[ , X_{el}, Y_e, 'm.', X_{el}, -Y_e, 'm.', X_{et}, Y_e, 'm.' ... \]
  \[ , X_{et}, Y_{\text{cut1}}, 'g.', X_{et}, Y_{\text{cut2}}, 'g.', ... \]
  \[ \text{axis equal} \]
% Opening panel-file
fid = fopen('squarenose_cor_single.p','w');
end

% Writing data to panel-file
fprintf(fid,'%i
',1); % Number of structures
fprintf(fid,'%i
',0);
fprintf(fid,'%1.15E %1.15E %1.15E
',0.5,0,0); % xshear, yshear
and main angle
fprintf(fid,'%i
',1); % Number of geometries
fprintf(fid,'%i
',ptotal); % Number of panels
fprintf(fid,'%1.15E
',1); % Geometry belongs to structure number
fprintf(fid,'%1.15E
',1); % Geometry chord length
fprintf(fid,'%1.15E %1.15E
',xt,0); % Trailing edge
fprintf(fid,'%1.15E %1.15E
',xl,0); % Leading edge
% Upper trailing edge
for i = 1:length(Ye)
fprintf(fid,'%1.15E %1.15E
',Xet(i),Ye(i));
end
% Upper chord
for i = 1:length(Yc)-1
fprintf(fid,'%1.15E %1.15E
',Xc(i+1),Yc(i+1));
end
% Upper leading edge
for i = 1:length(Ye)-1
fprintf(fid,'%1.15E %1.15E
',Xel(i+1),Ye(i+1));
end
% Additional data according to geometry
if geo == 1
% Lower chord
for i = 1:length(Yc)-1
fprintf(fid,'%1.15E %1.15E
',Xc(i+1),Yc(i+1));
end
elseif geo == 2
% Lower leading chord
for i = 1:length(Ycl)-1
fprintf(fid,'%1.15E %1.15E
',Xcl1(i+1),Ycl1(i+1));
end
% Leading cut
for i = 1:length(Ycut1)-1
fprintf(fid,'%1.15E %1.15E
',Xcut1(i+1),Ycut1(i+1));
end
% Trailing cut
for i = 1:length(Ycut2)-1
fprintf(fid,'%1.15E %1.15E
',Xcut2(i+1),Ycut2(i+1));
end
% Lower trailing chord
for i = 1:length(Ycl)-1
fprintf(fid,'%1.15E %1.15E
',Xcl2(i+1),Ycl1(i+1));
end
end

% Lower trailing edge
for i = 1:length(Ye)-2
  fprintf(fid,'%1.15E %1.15E \n',Xet(length(Ye)-i),-Ye(length(Ye)-i));
end
fclose(fid);

else
  \% Chosing number of panels for the different sides
  pe = ceil(Ledge/Lp);
  pc = ceil(h/2/Lp);
  pg1 = ceil(h/Lp);
  pg2 = ceil(h/2/Lp);
  pcl = ceil(B/4/Lp);
  pcut = ceil(Lcut/Lp);
  \% The offset generated by the gap
  offset = g/2;
  \% Creating general positions (x,y) for the panels
  Xel = linspace(-0.16*B-offset,0-offset,pe+1);
  Xet = linspace(1.16*B+offset,B+offset,pe+1);
  Xc1 = linspace(0-offset,B/2-offset,pc+1);
  Xc2 = linspace(B/2+offset,B+offset,pc+1);
  Xg1 = linspace(0.5-offset,0.5-offset,pg1+1);
  Xg2 = linspace(0.5+offset,0.5+offset,pg1+1);
  Ye = linspace(0,h/2,pe+1);
  Yc = linspace(h/2,h/2,pc+1);
  Yg = linspace(-h/2,h/2,pg1+1);
  \% Choice according to geometry type
  if geo == 1
    \% Total number of panels
    ptotal = 4*pe+4*pc+2*pg1;
    \% Plotting the geometry
    plot(Xc1,Yc,'r.',Xc2,Yc,'r.',Xcl,-Ye,'b.',Xc2,-Ye,'b.' ... 
      ,Xel,Ye,'m.',Xel,-Ye,'m.',Xg2,Yg,'m.' ... 
      ,Xet,Ye,'k.',Xet,-Ye,'k.',Xg1,Yg,'k.')
    axis equal
    \% Opening panel-file
    fid = fopen('squarenose_twin.p','w');
  elseif geo == 2
    \% Total number of panels
    ptotal = 4*pe+2*pc+2*pcl+2*pcut+2*pg2;
    \% Creating additional positions (x,y) for the panels for the cut
    Xg1 = linspace(0.5-offset,0.5-offset,pg2+1);
    Xg2 = linspace(0.5+offset,0.5+offset,pg2+1);
    Xcl1 = linspace(0-offset,B/4-offset,pc+1);
    Xcl2 = linspace(3*B/4+offset,B+offset,pc+1);
    Xcut1 = linspace(B/4+offset,B/2+offset,pcut+1);
    Xcut2 = linspace(B/2+offset,3*B/4+offset,pcut+1);
    Yg1 = linspace(0,h/2,pg2+1);
    Yg2 = linspace(h/2,0,pg2+1);
    Ycl1 = linspace(-h/2,-h/2,pc+1);
    Ycut1 = linspace(-h/2,0,pcut+1);
    Ycut2 = linspace(0,-h/2,pcut+1);
    \% Plotting the geometry
plot(Xc1,Yc,'r.',Xc2,Yc,'r.',Xcl1,Ycl,'b.',Xcl2,Ycl,'b.' ...
   ,Xel,Ye,'m.',Xel−Ye,'m.',Xg2,Yg2,'m.',Xcut1,Ycut1,'g.' ...
   ,Xcut2,Ycut2,'g.' ,Xet,Ye,'k.',Xet−Ye,'k.',Xg1,Yg1,'k.');
axis equal

% Opening panel-file
fid = fopen('squarenose.cor_twin.p', 'w');

% Writing general geometry data
fprintf(fid, '%i
', 1); % Number of structures
fprintf(fid, '%i
', 0);
fprintf(fid, '%1.15E %1.15E %1.15E
', 0.5, 0, 0); % xshear, yshear and main angle
fprintf(fid, '%i
', 2); % Number of geometries

% Writing data regarding first section
fprintf(fid, '%i
', ptotal/2); % Number of panels
fprintf(fid, '%i
', 1); % Geometry belongs to structure number
fprintf(fid, '%1.15E
', 0.5); % Geometry chord length
fprintf(fid, '%1.15E %1.15E %1.15E %1.15E
', 0.5−offset, 0);
% Trailing edge
fprintf(fid, '%1.15E %1.15E %1.15E
', xl, 0); % Leading edge
fprintf(fid, '%1.15E %1.15E %1.15E
', Xc1(length(Yc)−i),Yc(length(Yc)−i));

% Upper chord
for i = 0:length(Yc)−1
   fprintf(fid, '%1.15E %1.15E
', Xc1(i+1),−Yc(i+1));
end
% Upper leading edge
for i = 1:length(Ye)−1
   fprintf(fid, '%1.15E %1.15E
', Xel(length(Ye)−i),Ye(length(Ye)−i));
end
% Lower leading edge
for i = 1:length(Ye)−1
   fprintf(fid, '%1.15E %1.15E
', Xel(i+1),−Ye(i+1));
end
% Data according to geometry type
if geo == 1
   % Lower chord
   for i = 1:length(Yc)−1
      fprintf(fid, '%1.15E %1.15E
', Xc1(i+1),−Yc(i+1));
   end
   % Trailing gap edge
   for i = 1:length(Yg)−2
      fprintf(fid, '%1.15E %1.15E
', Xg1(length(Yg)−i),−Yg(length(Yg)−i));
   end
elseif geo == 2
   % Lower leading chord
   for i = 1:length(Ycl)−1
      fprintf(fid, '%1.15E %1.15E
', Xcl1(i+1),Ycl(i+1));
   end
   % Leading cut
   for i = 1:length(Ycut1)−1
      fprintf(fid, '%1.15E %1.15E
', Xcut1(i+1),Ycut1(i+1));
   end
D.1 Panel file generator

```matlab
% Trailing gap edge
for i = 1:length(Yg1) - 2
    fprintf(fid, '%1.15E %1.15E \n', Xg1(i+1), Yg1(i+1));
end

% Writing data regarding second section
fprintf(fid, '%i \n', ptotal/2); % Number of panels
fprintf(fid, '%i \n', 1); % Geometry belongs to structure number
fprintf(fid, '%1.15E \n', 0.5); % Geometry chord length
fprintf(fid, '%1.15E %1.15E \n', xt, 0); % Trailing edge
fprintf(fid, '%1.15E %1.15E \n', 0.5+offset, 0); % Leading edge
% Upper trailing edge
for i = 1:length(Ye)
    fprintf(fid, '%1.15E %1.15E \n', Xet(i), Ye(i));
end
% Upper chord
for i = 1:length(Yc) - 1
    fprintf(fid, '%1.15E %1.15E \n', Xc2(length(Yc) - i), Yc(length(Yc) - i));
end

% Data according to geometry type
if geo == 1
    % Leading gap edge
    for i = 1:length(Yg) - 1
        fprintf(fid, '%1.15E %1.15E \n', Xg2(length(Yg) - i), Yg(length(Yg) - i));
    end
    % Lower chord
    for i = 1:length(Yc) - 1
        fprintf(fid, '%1.15E %1.15E \n', Xc2(i+1), -Yc(i+1));
    end
else geo == 2
    % Trailing gap edge
    for i = 1:length(Yg2) - 1
        fprintf(fid, '%1.15E %1.15E \n', Xg2(i+1), Yg2(i+1));
    end
    % Leading cut
    for i = 1:length(Ycut2) - 1
        fprintf(fid, '%1.15E %1.15E \n', Xcut2(i+1), Ycut2(i+1));
    end
    % Lower leading chord
    for i = 1:length(Ycl) - 1
        fprintf(fid, '%1.15E %1.15E \n', Xcl2(i+1), Ycl(i+1));
    end
end
% Lower trailing edge
for i = 1:length(Ye) - 2
    fprintf(fid, '%1.15E %1.15E \n', Xet(length(Ye) - i), -Ye(length(Ye) - i));
end
fclose(fid);
```
D.2 Flutter routine

Description
The flutter routine that determines the critical wind speed for flutter $U_c$ based on the aerodynamic derivatives and the structural parameter and the dynamic properties.

Input
A text-file *.txt containing only numbers columns for the aerodynamic derivative in the format $[U_r\ H_1^*\ H_2^*\ A_1^*\ A_2^*]$.
A text-file *.txt containing the structural parameters and the dynamic properties.

Output
A pdf-file derivative.pdf showing the derivatives.
A pdf-file rootcurve.pdf showing the rootcurves, their intersection and the value of $U_c$.

---

```matlab
% Script with Theodorsen Flutter Routine
% used to determine the critical wind speed
% for flutter based on the aerodynamic
% derivatives from aero.
% Morten Plum DTU 042698 26-01-2010

clc
clear all

% Loading aerodynamic derivatives results
[file,path] = uigetfile(’*.txt’, ’Aerodynamic derivatives data file’);
aero = load([path file]);
Ur = aero(:,1);
H1 = aero(:,2); H2 = aero(:,3); H3 = aero(:,4); H4 = aero(:,5);
A1 = aero(:,6); A2 = aero(:,7); A3 = aero(:,8); A4 = aero(:,9);

% Correction by scaling forces to compare to Larsen1998c
H1 = 0.68*H1; H2 = 0.68*H2; H3 = 0.68*H3; H4 = 0.68*H4;
A1 = 0.68*A1; A2 = 0.68*A2; A3 = 0.68*A3; A4 = 0.68*A4;

% Loading structural data
[file,path] = uigetfile(’*.m’, ’Structural data file’);
run([path file])

% Coefficients Real Part Of Flutter Determinant
CR4 = 1*(r*B^2)/(m*B^4+e^2)+(r*B^4)/(I*A3)+(r^2*B^6)/(m*I)+H1-A2-A3-A4.

% CR3 = 2*za*(fa+r*B^2)/(fh*m)+H1+2*zh*(r*B^4)/I*A2;
```
D.2 Flutter routine  

\[ CR_2 = -(fa/fh)^2 - (fa/fh) \cdot \left( 1 - \frac{r \cdot B^4}{I} \cdot \frac{A_3}{A_2} - \frac{r \cdot B^2}{m} \cdot \left( \frac{fa/fh}{H_4} \right)^2 \cdot H_3 \right) \]

\[ CR_0 = (fa/fh)^2 \cdot \text{ones}(\text{length}(Ur),1) \]

% Coefficient Excluding Structural Damping — Initial Guess
\[ cR_2 = -(fa/fh)^2 - 1 \cdot \left( \frac{r \cdot B^4}{I} \cdot \frac{A_3}{A_2} - \frac{r \cdot B^2}{m} \cdot \left( \frac{fa/fh}{H_4} \right)^2 \cdot H_3 \right) \]

% Coefficients Imaginary Part Of Flutter Determinant
\[ CI_3 = \frac{r \cdot B^4}{I} \cdot \frac{A_2}{A_2} + \frac{r \cdot B^2}{m} \cdot H_1 + \frac{r^2 \cdot B^6}{m \cdot I} \cdot \left( \frac{H_4}{A_2} - \frac{A_4}{H_2} \cdot H_3 \right) \]

\[ CI_2 = -2 \cdot za \cdot \frac{fa/fh}{H_4} - 2 \cdot zh \cdot \left( \frac{r \cdot B^2}{m} \cdot H_4 \right) \]

\[ CI_1 = -\left( \frac{r \cdot B^4}{I} \right) \cdot \frac{A_2}{A_2} - \left( \frac{r \cdot B^2}{m} \right) \cdot \left( \frac{fa/fh}{H_4} \right)^2 \cdot H_4 \]

\[ CI_0 = 2 \cdot zh \cdot \left( \frac{fa/fh}{H_4} \right)^2 + 2 \cdot za \cdot \frac{fa/fh}{H_4} \cdot \text{ones}(\text{length}(Ur),1) \]

% Initial Guess, Real Root
\[ XR = \sqrt{\left( -cR_2 + \sqrt{cR_2^2 - 4 \cdot CR_4 \cdot CR_0} \right)} \]

% Initial Guess, Imaginary Root
\[ XI = \sqrt{-\frac{CI_1}{CI_3}} \]

% Root Curves of Flutter Determinant
\[ XR = \text{zeros}(1,\text{length}(Ur)); \]
\[ XI = \text{zeros}(1,\text{length}(Ur)); \]

for \( i = 1: \text{length}(Ur) \)
\[ Fr = @(x)CR_4(i) \cdot x^4 + CR_3(i) \cdot x^3 + CR_2(i) \cdot x^2 + CR_0(i); \]
\[ XR(i) = \text{fsolve}(Fr,XR(i)); \]
\[ Fi = @(x)CI_3(i) \cdot x^3 + CI_2(i) \cdot x^2 + CI_1(i) \cdot x + CI_0(i); \]
\[ XI(i) = \text{fsolve}(Fi,XI(i)); \]
end

% Locating point after intersection XR, XI
if XR(1) > XI(1)
\[ index = \text{find}(XI>XR,1); \]
else
\[ index = \text{find}(XR>XI,1); \]
end

% Error messages if no intersection is present
if isempty(index)
\[ \text{fprintf(‘

No intersection between XR and XI

’)} \]
\[ \text{fprintf(‘

’)} \]
else
% Coordinates of line points before and after intersection
\[ x1 = Ur(\text{index}-1); y1 = XR(\text{index}-1); \]
\[ x2 = Ur(\text{index}); y2 = XR(\text{index}); \]
\[ x3 = Ur(\text{index}-1); y3 = XI(\text{index}-1); \]
\[ x4 = Ur(\text{index}); y4 = XI(\text{index}); \]

% Determining the point of intersection
\[ \text{dummy1} = \text{det}([x1 \ y1; x2 \ y2]); \]
\[ \text{dummy2} = \text{det}([x3 \ y3; x4 \ y4]); \]
\[ \text{Urc} = \text{det}([\text{dummy1} x1-x2; \text{dummy2} x3-x4])/\text{det}([x1-x2 y1-y2; x3-x4 y3-y4]); \]
\[ \text{Xc} = \text{det}([\text{dummy1} y1-y2; \text{dummy2} y3-y4])/\text{det}([x1-x2 y1-y2; x3-x4 y3-y4]); \]

% Critical Wind Speed For Flutter
\[ Uc = Xc \cdot Ur \cdot fh \cdot B; \]

% Flutter Frequency
fc = fh*Xc;
end

figure(1)
clf
subplot(1,2,1)
hold on, box on
plot(Ur,H1,Ur,H2,Ur,H3,Ur,H4)
legend('H1', 'H2', 'H3', 'H4', -1)
xlabel('Ur')
subplot(1,2,2)
hold on, box on
plot(Ur,A1,Ur,A2,Ur,A3,Ur,A4)
legend('A1', 'A2', 'A3', 'A4', -1)
xlabel('Ur')

figure(2)
clf
hold on, box on
plot(Ur,XR,Ur,XI)
if ~isempty(index)
    plot(Urc,Xc,'ro')
    legend('XR', 'XI', '(Urc,Xc)')
    text(min(Ur)+2,(max(XR)+min(XR))/2, ['Uc = ', num2str(Uc)])
else
    legend('XR', 'XI')
end
xlabel('Ur')

[file,path] = uiputfile('derivatives.pdf;rootcurves.pdf','Save plots');
if iscell(file)
    print('-f1','-dpdf', [path char('derivatives')])
    print('-f2','-dpdf', [path char('rootcurve')])
end

close all
D.3 Geometry viewer

Description
Plotting a geometry based on a panel-file as used in DVMFLOW

Input
A panel-file *.p containing the information regarding the geometry

Output
A pdf-file geometry.pdf showing the geometry

```matlab
% Script for viewing the panel file used in DVMFlow. File example: "messina.p"

% Morten Plum DTU
s042698 20−01−2010

clc
clear all
close all

% Choosing a panel file to plot
[file,path] = uigetfile(‘*.p’, ‘Pick an panel-file’);

% Opening and reading panel file
fid = fopen([path file]);
data = fscanf(fid,’%f’);
fclose(fid);

% Number of structures in the file
Nstruct = data(1);

% Adjusting the counter according to number of structures
for i = 1:Nstruct
    count = 4*i+3;
    Ngeo = data(count-1);
end

figure(1)
cf
hold on, box on

% Looping over each geometry
for i = 1:Ngeo
    % Number of panels in geometry
    Npanel = data(count);
    % The positions of the panels in the geometry
    xy = reshape(data(count+7:count+6+2*Npanel),2,[]);
    % Plot the geometry
    plot([xy(:,1);xy(1,1)],[xy(:,2);xy(1,2)])
    % Update the counter
    count = count+6+2*Npanel + 1;
end
axis equal
```
% Saving the plot
[file,path] = uiputfile('geometry.pdf','Save plot as');
print('-f1','-dpdf',[path file])
close all