EXPERIMENTAL INVESTIGATION OF FREQUENCY RESPONSE OF UNSTEADY PRESSURE SENSITIVE PAINT AND APPLICATION FOR AN OSCILLATING AIRFOIL

by

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Technical University of Denmark

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DTU

11th of January 2008
EXPERIMENTAL INVESTIGATION OF FREQUENCY RESPONSE OF UNSTEADY PRESSURE SENSITIVE PAINT AND APPLICATION FOR AN OSCILLATING AIRFOIL

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Preface

This Master’s thesis has been completed for the Fluid Mechanics section of the Department of Mechanical Engineering of the Technical University of Denmark under supervision of Prof. Jens Nørkær Sørensen.

The actual work has taken place at the German Aerospace Center, DLR, in Göttingen, in the Experimental Methods section, in the Pressure Sensitive Paint (PSP) group led by Dr. Christian Klein who also acted as the primary supervisor at the DLR.

The thesis has been completed in the period from February 2007 to January 2008 within the framework of the Unsteady PSP Project of the PSP group, and amounts to 40 ECTS credits.
Acknowledgments

First and foremost I would like to thank my official supervisors, Prof. Jens N. Sørensen and Dr. Christian Klein, for making this whole project possible, and for help and guidance.

No lesser thanks goes to Dr. Ulrich Henne and Dr. Werner Sachs who has also acted as supervisors for me, and have been of invaluable help, as well as the rest of the PSP group, Dr. Uwe Fey, Dr. Yasuhiro Egami and Dr. Walther Beck, who have always been helpful and excellent company.

In general I would like to thank all people at DLR Göttingen, with whom i have been directly or indirectly involved.

Finally I would like to thank my girlfriend Pernille, my family, and my friends in Denmark and Germany, for their consistent support.

Jakob Borbye (s021936)

Kongens Lyngby
11th January 2008
Abstract

A new experimental setup for accurate experimental determination of the frequency response of pressure sensitive paints (PSP) is presented. By means of a mechanical shaker connected to a pressure chamber the setup realizes high-quality sinusoidal pressure variations for frequencies ranging from less than 0.1 Hz to several hundreds of Hz with pressure amplitudes up to 3 kPa. An optical system for detection of fluorescent light has been designed and developed as part of the work. It is demonstrated that the measurement system including the PSP is adequately described as a linear time-invariant system, allowing for characterization of the paint by a transfer function.

Unsteady characterization of various formulations of PSP based on the luminophore platinum tetra(pentafluorophenyl)porphine (PtTFPP) in a fluoroacrylic polymer (FIB) was carried out for frequencies $f = 0.1 - 120$ Hz. The influence on frequency response from paint layer thickness was measured and reasonably well modeled with a PSP diffusion model extended to account for internal light attenuation in the paint layer. The influence of paint porosity is tested by adding from 0 to 80 mass % TiO$_2$ particles to the PSP. The frequency response is shown to be similar with respect to a characteristic time, which ranged from the order of seconds for no filler to the order of milliseconds for 80% TiO$_2$.

In a wind tunnel campaign in the the DNW-TWG in collaboration with the DLR institute of Aeroelasticity, a 2D airfoil coated with a fast-responding PSP was investigated. The airfoil was subject to 30 Hz forced pitching oscillation at a Mach number of 0.72 and an angle of attack of $1.12° \pm 0.6°$. Phase-locked pressure field measurements were compared to conventional pressure measurements for a chordal section. Correction of the pressure time series was conducted, by computing the Fourier series, truncating it at $f = 120$ Hz, and correcting phase and amplitude by means of the transfer function determined or the PSP.

The correction was seen to improve the pressure principally, if only modestly. The success of the method relies on that the truncated Fourier transform is a good fit to the data, which was not the case for the present measurements, which contained high-frequency components from traveling shock waves.
# Nomenclature

## Scalar Quantities

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>$P$</td>
<td>pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
<td>K</td>
</tr>
<tr>
<td>$I$</td>
<td>intensity</td>
<td>Cd</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength</td>
<td>m</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency</td>
<td>rad/s</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>$k$</td>
<td>rate constant</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
<td>s</td>
</tr>
<tr>
<td>$\phi$</td>
<td>phase</td>
<td>rad or °</td>
</tr>
<tr>
<td>$c$</td>
<td>Airfoil chord</td>
<td>m</td>
</tr>
<tr>
<td>$S$</td>
<td>Molecular energy state</td>
<td>-</td>
</tr>
<tr>
<td>$h$</td>
<td>Paint layer thickness</td>
<td>m</td>
</tr>
<tr>
<td>$\hbar$</td>
<td>Plank’s constant</td>
<td>6.63 · 10$^{-34}$ J/s</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Quantum yield</td>
<td>-</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Error norm</td>
<td>-</td>
</tr>
<tr>
<td>$N$</td>
<td>Integer</td>
<td>-</td>
</tr>
<tr>
<td>$(x, y, z)$</td>
<td>Cartesian spatial coordinates</td>
<td>m</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle of attack</td>
<td>rad or °</td>
</tr>
<tr>
<td>$J$</td>
<td>Thickness-specific Intensity</td>
<td>Cd/m</td>
</tr>
<tr>
<td>$Re$</td>
<td>Chord-based Reynolds number</td>
<td>-</td>
</tr>
<tr>
<td>$Ma$</td>
<td>Mach number</td>
<td>-</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Characteristic time</td>
<td>s</td>
</tr>
<tr>
<td>$D$</td>
<td>Diffusivity</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Mole fraction</td>
<td>-</td>
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## Operators and Functions

<table>
<thead>
<tr>
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<th>Description</th>
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<tbody>
<tr>
<td>$\delta x$</td>
<td>Dirac/Kronecker delta function</td>
</tr>
<tr>
<td>$\partial_i x$</td>
<td>Partial differentiation with respect to $x$</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>Phase-resolved/ensemble average of $x$</td>
</tr>
<tr>
<td>$\overline{x}$</td>
<td>Time average of $x$</td>
</tr>
<tr>
<td>$&lt; x &gt;$</td>
<td>Spatial average of $x$</td>
</tr>
<tr>
<td>$[x]$</td>
<td>Concentration or population of $x$</td>
</tr>
<tr>
<td>$\mathcal{F}[x]$</td>
<td>Fourier transform of $x$</td>
</tr>
<tr>
<td>$(h \otimes x)(t)$</td>
<td>Convolution of $x(t)$ with $h(t)$</td>
</tr>
<tr>
<td>$x \ast y$</td>
<td>Cross-correlation between $x$ and $y$</td>
</tr>
<tr>
<td>$R(x, y)$</td>
<td>Correlation coefficient between $x$ and $y$</td>
</tr>
<tr>
<td>$\Re(x)$</td>
<td>Real part of $x$</td>
</tr>
<tr>
<td>$\Im(x)$</td>
<td>Imaginary part of $x$</td>
</tr>
<tr>
<td>$f(T, P)$</td>
<td>Calibration function</td>
</tr>
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## Sub- and superscripts

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$x_{\text{ref}}$</td>
<td>Reference value</td>
</tr>
<tr>
<td>$x_{\text{wo}}$</td>
<td>Wind-off measurement</td>
</tr>
<tr>
<td>$x_r$</td>
<td>Intensity monitoring component of paint (&quot;r&quot; for &quot;red&quot;) / radiative</td>
</tr>
<tr>
<td>$x_b$</td>
<td>Pressure sensitive component of paint (&quot;b&quot; for &quot;blue&quot;)</td>
</tr>
<tr>
<td>$x_{\text{ex}}$</td>
<td>Excitation</td>
</tr>
<tr>
<td>$x_{\text{nr}}$</td>
<td>Non-radiative</td>
</tr>
<tr>
<td>$x_q$</td>
<td>Quenching</td>
</tr>
<tr>
<td>$x_s$</td>
<td>Sampling</td>
</tr>
<tr>
<td>$x^*$</td>
<td>Excited State / Non-dimensional</td>
</tr>
<tr>
<td>$x^\dagger$</td>
<td>Raw (un-manipulated) image</td>
</tr>
<tr>
<td>$x^\dagger$</td>
<td>Raw (un-manipulated) image / Excited State / Non-dimensional</td>
</tr>
<tr>
<td>$x^{(j)}$</td>
<td>$j$ first terms of Fourier series expansion of $x$</td>
</tr>
</tbody>
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### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>APD</td>
<td>Avalanche Photo Diode</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge Coupled Device</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DTFT</td>
<td>Discrete-Time Fourier Transform</td>
</tr>
<tr>
<td>DLR</td>
<td>Deutsches Zentrum für Luft- und Raumfahrt (German Aerospace Center)</td>
</tr>
<tr>
<td>DNW</td>
<td>German-Dutch Wind Tunnels</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree Of Freedom</td>
</tr>
<tr>
<td>DTU</td>
<td>Danmarks Tekniske Universitet (Technical University of Denmark)</td>
</tr>
<tr>
<td>EMCCD</td>
<td>Electron Multiplication Charge Coupled Device</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FOS</td>
<td>First Order System</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
</tr>
<tr>
<td>iPSP</td>
<td>Unsteady (&quot;instationäres&quot;) PSP</td>
</tr>
<tr>
<td>LED</td>
<td>Light Emitting Diode</td>
</tr>
<tr>
<td>PD</td>
<td>Photo Diode</td>
</tr>
<tr>
<td>PMT</td>
<td>Photo Multiplier Tube</td>
</tr>
<tr>
<td>PSP</td>
<td>Pressure Sensitive Paint</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>TSP</td>
<td>Temperature Sensitive Paint</td>
</tr>
<tr>
<td>ToPas</td>
<td>Three-dimensional Optical Pressure Analysis System</td>
</tr>
<tr>
<td>TWG</td>
<td>Transonic Wind tunnel, Göttingen</td>
</tr>
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1. Introduction

Surface pressure measurements are of fundamental importance in aerodynamic testing. For both determination of aerodynamic loads on structures, characterizing flow topology as e.g. separation and impinging shock waves, and for providing data for validation of CFD codes, knowledge of surface pressures are invaluable. The conventional way of measuring surface pressures is with pressure taps in the model surface connected to pressure transducers either directly or through tubing. These techniques have are very well known and state-of-the-art with respect to accuracy and response time. However when high spatial resolution is required, the use of discrete-point pressure taps becomes extremely expensive and time-consuming. Also, for some model geometries, e.g. very thin airfoils, the embedding of pressure taps is simply not practically realizable.

Since the 1980’s Pressure Sensitive Paint (PSP) measurement techniques, have become an alternative to conventional pressure taps for steady pressure distributions [1]. A PSP relies on microscopic photophysical processes in an oxygen-permeable paint layer. The paint incorporates sensor molecules, whose luminescence is depending on the oxygen concentration in the environment. The polymer acts as a matrix containing the sensor molecules, while simultaneously allowing for oxygen to diffuse into the polymer and interact with the sensor molecules. By these mechanisms a emitted luminescent intensity from a PSP layer is related to the pressure at the paint surface. These mechanisms are explained in more detail in the following section.

The intensity emitted from the PSP can be measured with a light detection system, and due to the microscopic scale of the PSP mechanisms, it is in practice only the light detection system which limits the spatial resolution of PSP. If for instance the intensity distribution from a model coated with PSP is acquired with a CCD camera, every pixel becomes effectively a pressure sensor. This unsurpassed spatial resolution makes PSP a true sensor for pressure distributions, which offers unique possibilities. An example of this is demonstrated in [2], where the full surface pressure field of an advanced 3D geometry (a fighter aircraft) was determined. The result was of sufficient quality to be used for quantitative conclusions, since the integrated lift force was predicted with an error of 5% compared to simultaneous force balance measurements. Simultaneously PSP provides a visualization of the full pressure field, providing insights about the flow which no other pressure measurement technique is currently able to give. Amongst other advantages of PSP over pressure taps is that much time and money for instrumentation of the model is saved, as the main part of the instrumentation is in essentially applied with a spray gun.
Due to its obvious virtues, it natural to attempt to extend the application field of this
technique to the investigation of unsteady flow phenomena. The potential benefit for a
range of investigations of e.g. aeroelastics, turbo machinery and rotor craft, is huge.

For this reason, DLR initiated in 2005 an internal Unsteady PSP project running until
2008, with the goal of developing the required hard- and software for industrial
unsteady PSP measurements with a temporal resolution of better than 1 µs and a
sampling rate of up to 100 Hz. The unsteadiness of the measurements influences all
aspects of the measurement systems, including cameras, illumination sources, control,
data acquisition and analysis procedures. Most importantly it includes the PSP itself.
A formulation was developed by DLR to provide fast response, realized by increasing
the porosity of the paint, which decreases the response time to pressure changes.

As any unsteady sensor, the response characteristics of unsteady PSP should be must
be accurately determined in order to be able to interpret and eventually correct the
results. In the present work a dedicated setup for determination of frequency response
of PSP, developed as part of the unsteady PSP project, is described and used conduct
parametric investigations for unsteady PSP. The frequency response of an unsteady
PSP was determined, and the result used for unsteady correction of wind tunnel
measurements made with the same PSP. This is to the authors knowledge the first
time the frequency response of porous PSP has been measured and used for
quantitative correction of experimental data.

The outline of the thesis is the following:

Chapter 2 lays out the basic theory needed to understand the steady and unsteady
behavior of PSP, and the analysis applied for PSP characterization.

Chapter 3 describes the hardware of the calibration setup along with the PSP itself.

Chapter 4 regards the measurement and analysis methodology, and presents
calculations and preliminary measurements, to show that the measurement system and
the paint is properly described by the notion of a linear time-invariant system.

Chapter 5 presents results for a parametric investigation of variations of the unsteady
PSP formulation. The influence of thickness and porosity is measured and modeled,
and the PSP used for wind tunnel tests characterized.

Chapter 6 describes the second wind tunnel experiment carried out as part of the
unsteady PSP project. The measurement system, and analysis for unsteady PSP is
described, and measurement results corrected by means of the PSP frequency
response found in chapter 5.
2. Background and Theory

In this chapter the fundamental principles of pressure sensitive paints (PSP) are explained. For a more detailed review of PSP technique in general the reader is referred to good introductory passages in [1], [3] and [4]. The notion of a linear time-invariant system and a transfer function, and models for unsteady PSP response are presented along with an overview of the literature on unsteady PSP characterization.

2.1. Basic Principles of Pressure Sensitive Paint

PSP’s are based on the principles of photoluminescence and (oxygen) quenching. A molecule which can be excited to a higher energy state by absorbing a photon, and subsequently decay into a lower state by emission of a photon, is called a luminophore, and its energy states can be visualized by a Jablonsky energy-level diagram as seen on figure Fig. 2.1 on the following page. Each of the electronic energy states ($S_0$, $S_1$ and $T_1$) have several degenerates which are vibrational and rotational states. The process of photoluminescence can be be visualized in the diagram as follows: A photon with frequency $\nu_{ex}$ is absorbed by a luminophore in the ground state $S_0$ and causes the transition to the excited state $S_2$.

$$S_0 + h\nu_{ex} \xrightarrow{k_{ex}} S_2 \quad (2.1)$$

where $h$ is Planck’s constant, and $k_{ex}$ the rate constant with which the process takes place. This process is called photo excitation and is represented by an "absorption" arrow on the Jablonsky diagram. A molecule in an excited state will eventually fall back to the ground state, i.e. the excited state will deactivate. This deactivation can happen by a variety of radiative and non-radiative processes. For a luminophore, the excited molecule will after internal conversion and relaxation typically end in a state from which radiative decay is possible. Radiative decay is the reverse of photoexcitation, for instance

$$S_1 \xrightarrow{k_r} S_0 + h\nu_{lum} \quad (2.2)$$

(See the "fluorescence" arrow Fig. 2.1 on the next page). This process is called luminescence and in combination with photoexcitation, photoluminescence. The $k_r$ is the radiative deactivation rate constant and $\nu_{lum}$ the wavelength of the luminescence. Remark that due to the intermediate energy losses, $\nu_{lum} < \nu_{ex}$. Deactivation by
2.1 Basic Principles of Pressure Sensitive Paint

Figure 2.1: Example of a Jablonsky energy-level diagram [1]. The horizontal lines represent energy states, and the arrows transitions between these states.
2.1 Basic Principles of Pressure Sensitive Paint

2 Background and Theory

non-radiative processes encompasses a variety of effects including internal conversion, intersystem crossing, vibrational relaxation and quenching [1]. In this context quenching, which is a collective term for environmentally induced deactivation of energy states, is the process of interest. For aerodynamic applications the two most interesting quenching mechanisms are oxygen quenching and thermal quenching, which are the basic mechanisms of pressure and temperature sensitive paints respectively.

The mechanism of oxygen quenching is that the excited luminophore is deactivated by transferring its energy to an oxygen molecule, i.e.

\[ S_1 + O_2 \rightarrow S_0 + O_2^* \]  (2.3)

where the asterisk denotes an excited state. A first-order kinetic model for the population of the excited state of a luminophore, \([S_1]\), is given in [1] as

\[ \partial \frac{[S_1]}{t} = k_{ex}[S_0] - (k_r + k_{nr} + k_{O_2}[O_2])[S_1] \]  (2.4)

where the \(k_{ex}\), \(k_r\), \(k_{nr}\), \(k_{O_2}\) are rate constants for excitation and deactivation by radiative, non-radiative and oxygen quenching respectively. The rate constants are in general temperature-dependent of the form

\[ k = k_0 + k_1 \exp\left(\frac{E}{RT}\right) \]  (2.5)

due to the Arrhenius relation [1]. The values of \(k_0\), \(k_1\) and \(E\) are constants depending on the actual deactivation mechanism.

2.1.1. Steady PSP and Calibration Function

A PSP consists of luminophore molecules contained in a supporting matrix, typically a polymer, which must be permeable to oxygen in order to allow oxygen to interact with the luminophore, and sufficiently transparent to allow the transmission of incident and luminescent light. On Fig. 2.2 on the following page such a PSP layer is sketched.

Assuming that the system on is in a steady state, the population of each state of the luminophores is constant, in particular \(\partial_t[S_1] = 0\) in 2.4. The quantum yield, \(\Phi\) is the ratio of luminescence rate to excitation rate:

\[ \Phi = \frac{k_r[S_1]}{k_{ex}[S_0]} = \frac{k_r}{k_r + k_{nr} + k_{O_2}[O_2]} = \frac{I}{I_{ex}} \]  (2.6)

where \(I\) is the luminescent intensity and \(I_{ex}\) the excitation intensity. Evaluating the quantum yield at vacuum conditions ([O_2] = 0) and at a reference oxygen concentration, and dividing the resulting expressions gives

\[ \frac{I_0}{I} = 1 + \frac{k_{O_2}}{k_r + k_{nr}}[O_2] \]  (2.7)

\(^1\)Other gases than oxygen can be used as quenchers for PSP, but due to the fact that oxygen is present in atmospheric air (21 volume %) and that it has good quencher properties [3], oxygen is the quencher for the vast majority of PSP’s.
2.1 Basic Principles of Pressure Sensitive Paint

Using Henry’s law [5], it is assumed that the oxygen concentration in the polymer, \( n = [O_2] \), is proportional to the oxygen partial pressure at the surface.

\[
n = \theta \chi_{O_2} P
\]  

(2.8)

where \( \theta \) is the oxygen solubility in the given polymer, \( \chi_{O_2} \) the mole fraction of \( O_2 \) in the gas, and \( P \) the total pressure. With these assumptions (2.7) becomes

\[
\frac{I_0}{T} = 1 + g(T) P \quad \text{where} \quad g(T) = \frac{k_{O_2}}{k_r + k_{nr}} \theta_{O_2}
\]  

(2.9)

since the rate constants and the oxygen solubility are assumed to depend only on temperature. It is inconvenient to have the vacuum condition appear the formula and thus a reference condition \((P_{\text{ref}}, I_{\text{ref}})\) is introduced. Equation (2.9) is evaluated with at general and reference conditions respectively, and the resulting expressions divided, giving

\[
\frac{I_{\text{ref}}}{P} = A(T) + B(T) \frac{P}{P_{\text{ref}}}
\]  

(2.10)

where \( \frac{I_{\text{ref}}}{P_{\text{ref}}} \) is the ratio between the emitted intensity at a reference pressure, and at the considered pressure, \( P \) is the pressure, \( T \) is the temperature and \( A(T) \) and \( B(T) \) are temperature-dependent coefficients. This is known as the Stern-Volmer equation, and is a reasonable description of the behavior of most PSP’s. Introducing the non-dimensional variables

\[
P^* = \frac{P}{P_{\text{ref}}}, \quad I^* = \frac{I}{I_{\text{ref}}}, \quad T^* = \frac{T}{T_{\text{ref}}}
\]  

(2.11)
the Stern-Volmer relation can be written in non-dimensional and slightly rearranged form

\[ I^* = f(T^*, P^*) = \frac{1}{a_1(T^*) + b_1(T^*)P^*} \]  

(2.12)

The right hand side of (2.10) is the calibration function for the paint, \( f(T^*, P^*) \) characterized by the value of the coefficients \( a_1(T^*) \) and \( b_1(T^*) \). In the following the term calibration function is used for a generic function establishing the relationship between the intensity and the pressure, i.e. in general

\[ I^* = f(T^*, P^*) \quad \text{and} \quad P^* = f^{-1}(T^*, I^*) \]  

(2.13)

Since the Stern-Volmer is not sufficiently accurate for quantitative measurements with most real paints, the standard form of the calibration function used for practical measurements is:

\[ f(T^*, P^*) = I^* = \left[ \sum_{i,j=0}^2 a_{ij} T^{i*} P^{j*} \right]^{-1} \]  

(2.14)

where the coefficients \( a_{ij} \) are temperature independent and define the calibration function. These coefficients are determined by measuring a number of intensity measurements, \( N \), for different known temperatures and pressures, and setting up the system of equations:

\[
\begin{bmatrix}
1 & T_1^* & T_1^{*2} & P_1^* & T_1^* P_1^* & T_1^{*2} P_1^* & P_1^{*2} & T_1^* P_1^{*2} & T_1^{*2} P_1^{*2} \\
1 & T_2^* & T_2^{*2} & P_2^* & P_2^{*2} & T_2^* P_2^{*2} & P_2^{*2} & P_2^* P_2^{*2} & T_2^* P_2^{*2} \\
1 & T_3^* & T_3^{*2} & P_3^* & P_3^{*2} & T_3^* P_3^{*2} & P_3^{*2} & P_3^* P_3^{*2} & T_3^* P_3^{*2} \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
1 & T_N^* & T_N^{*2} & P_N^* & P_N^{*2} & T_N^* P_N^{*2} & P_N^{*2} & P_N^* P_N^{*2} & T_N^* P_N^{*2}
\end{bmatrix}
\begin{bmatrix}
a_{00} \\
a_{01} \\
a_{02} \\
a_{10} \\
a_{11} \\
a_{12} \\
a_{20} \\
a_{21} \\
a_{22}
\end{bmatrix}
= \begin{bmatrix}
I_1^{*-1} \\
I_2^{*-1} \\
I_3^{*-1} \\
\vdots \\
I_N^{*-1}
\end{bmatrix}
\]  

(2.15)

This system is an underdetermined system of the form

\[ Ax = b \]  

(2.16)

As such it cannot be solved in the exact sense, yet in the least-squares sense for which the solution is

\[ x = (A^T A)^{-1} A^T b \]  

(2.17)
This yields the coefficients which are inserted into the calibration function (2.14). For this, it is obtained by solving (2.14) with respect to \( p \), yielding

\[
f^{-1}(T^*, I^*) = -b_1 \pm \sqrt{b_1^2 - 4b_2(b_0 - I^{*-1})} \quad \text{where} \quad b_j = \sum_{i=0}^{2} a_{ij}T^{si}
\]

(2.18)

where the physical solution is that for which \( P \) lies within the range of calibration pressures.

### 2.1.2. Binary PSP

When the calibration function \( f(P, T) \) is known, pressure measurements can be conducted on a model. The intensity, \( I \), is measured and if the temperature \( T \) is known through measurements or through assumption, the pressure, \( P \), can be determined. Unlike for a calibration, where conditions are controlled, these quantities are generally all function the position on the model surface \( r \) in a real measurement situation, which is emphasized by writing \( P^* = P^*(r), T^* = T^*(r) \) and \( I^* = I^*(r) \). For the measured intensity \( I^*(r) \), the following equation holds

\[
I^*(r) = k_0I_{ex}(r)f[T^*(r), P^*(r)]g(r)
\]

(2.19)

which expresses that apart from being proportional to the calibration function, \( f[T^*(r), P^*(r)] \), the measured intensity is also linearly depending on the received intensity from the excitation light source at location \( r \), \( I_{ex}(r) \), and on a function \( g(r) \) which describes the intensity dependence of spatial inhomogeneities in the sensor system. To be more specific \( g(r) \) can be expanded:

\[
g(r) = n_{lum}(r) \cdot \Theta [h(r)] \cdot \Phi (r)
\]

(2.20)

where \( n_{lum}(r) \) is the local concentration of sensor molecules, \( \Theta [d(r)] \) is a function of the local paint layer thickness \( h(r) \) and \( \Phi (r) \) is the detection efficiency of the emitted light from location \( r \). As one can imagine, these dependencies are quite complex and generally not known. To solve the problem of the many unknowns in (2.19), a measurement is made at ambient pressure and temperature. This is known as the wind-off measurement denoted with the subscript \( wo \). For the wind-off measurement the spatial variations in pressure and temperature are eliminated, i.e. \( P^*(r) = P_{wo}^* \) and \( T^*(r) = T_{wo}^* \). By evaluating (2.19) for the wind-on and wind-off condition respectively, and dividing the expressions, the result is

\[
\frac{I_{wo}^*(r)}{I^*(r)} = \frac{I_{ex,wo}(r)}{I_{ex}(r)} \cdot \frac{f(T_{wo}^*, P_{wo}^*)}{f[T^*(r), P^*(r)]}
\]

(2.21)

The spatial intensity field, \( f[T(r), P(r)] \) is isolated and inserted into (2.13), to yield the equation for the pressure:

\[
P(r) = f^{-1} \left( T(r), \frac{I_{ex,wo}(r)}{I_{ex}(r)} \cdot \frac{I^*_w(r)}{I_{wo}^*(r)} \cdot f(T_{wo}^*, P_{wo}^*) \right)
\]

(2.22)
For the right hand side of this equation, only \( \frac{I_{\text{ex,wo}}(r)}{I_{\text{ex}}(r)} \) is unknown. Keeping the illumination constant between the wind-off and wind-on measurements would result in a ratio of 1, however in practice this is hard to obtain due to model movement and light source instability. This is the reason for using binary PSP. A binary PSP contains a second luminophore, which is insensitive to pressure, and termed the intensity monitor or reference component. The idea is to use the intensity from the reference component, \( I_r^* \), as a correction, since it has all the dependencies of (2.19) and (2.20), except for the calibration function. Using the above procedure for the reference component therefore yields the result

\[
\frac{I_r^*(r)}{I_{r,\text{wo}}^*(r)} = \frac{I_{\text{ex}}(r)}{I_{\text{ex,wo}}(r)}
\]  

(2.23)

By inserting this result into the pressure equation (2.22), it is expressed in terms of measured quantities:

\[
P^*(r) = f^{-1}(T^*(r), q(r) \cdot f(T_{\text{wo}}^*, P_{\text{wo}}^*)) \quad \text{where} \quad q(r) = \frac{I_{r,\text{wo}}^*(r)}{I_r^*(r)} \cdot \frac{I^*(r)}{I_{\text{wo}}^*(r)}
\]  

(2.24)

In general there is a distinction between wind-off and reference conditions. If it is practical, wind-off condition can be chosen as reference condition, in which case \( f(T_{\text{wo}}, P_{\text{wo}}) = 1 \).

A sketch of a binary PSP setup is seen on Fig. 2.3. The excitation light source should have a wavelength matching the absorption of the luminophores, which should be as close to each other as possible. To provide sufficient excitation it should be provide
high intensity, and it should preferably be stable, even though this requirement is relaxed somewhat when using binary PSP. The luminescence from the two luminophores should be at different wavelengths, such that the signals can be separated with optical filters as seen on Fig. 2.3 on the preceding page. The light detectors are typically CCD cameras, which should generally have as high sensitivity and dynamic range as possible. Optical filters are used in front of the light detectors to separate pressure and reference luminescence.

### 2.2. Linear Time-invariant Systems and Frequency Response

In this context a dynamical system can be described as

\[ y(t) = f(x(t)) \]  \hspace{1cm} (2.25)

where \( x(t) \) is the input signal, \( y(t) \) the output signal and \( f \) the system operator or simply the system. A subset of dynamical systems are the linear and time-independent (LTI). For this class of systems the analysis is much simpler and elegant than in the general case. The following is covered in more detail in appendix App. A.1 on page 98. Consider an input signal composed of \( n \) superimposed components, \( x(t) = \sum_n x_n(t) \).

The system \( f \) is linear if

\[ f \left( \sum_n k_n x_n(t) \right) = \sum_n k_n f(x_n(t)) \]  \hspace{1cm} (2.26)

where \( k_n \) are constants and time-independent if the time is appear does not appear explicitly in the expression in \( f \), other than in the form of \( x(t) \). If the system is a LTI system it can be described completely in the time domain by its impulse response, \( h(t) \), and in the frequency domain by its transfer function, \( H(\omega) \). In Tab. 2.1 on the following page the relationship between these quantities is shown.

The goal of the whole analysis is to determine \( h(t) \). Then for an experiment where the output signal \( y(t) \) is measured, \( x(t) \) can be found by solving the equation

\[ y(t) = (h \otimes x)(t) \]  \hspace{1cm} (2.27)

If the output signal \( y(t) \) is periodic and bounded, it has a Fourier transform, and the (2.27) can be solved in the frequency domain (see Tab. 2.1 on the next page):

\[ x(t) = \mathcal{F}^{-1} \left[ \frac{Y(\omega)}{H(\omega)} \right] \]  \hspace{1cm} (2.28)

Determining the transfer function is therefore the key to resolving the original signal. At this point it should be mentioned that in practice discrete versions of these
Table 2.1: Connection between time and frequency domain, impulse and frequency response

<table>
<thead>
<tr>
<th>Time domain</th>
<th>output</th>
<th>input</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(t)$</td>
<td>$h(t)$</td>
<td>$x(t)$</td>
</tr>
<tr>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency domain</th>
<th>$Y(\omega)$</th>
<th>$H(\omega) \cdot X(\omega)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\downarrow$</td>
<td>Fourier transform</td>
<td></td>
</tr>
<tr>
<td>$\uparrow$</td>
<td>Inverse Fourier transform</td>
<td></td>
</tr>
<tr>
<td>$\cdot$</td>
<td>Multiplication</td>
<td></td>
</tr>
<tr>
<td>$\otimes$</td>
<td>Convolution</td>
<td></td>
</tr>
<tr>
<td>$h(t)$</td>
<td>Impulse response</td>
<td></td>
</tr>
<tr>
<td>$H(\omega)$</td>
<td>Transfer function</td>
<td></td>
</tr>
</tbody>
</table>

Transforms will be used and that the success of the method when used for a measured and discretized signal including noise will depend on the signal quality, sampling rate etc. This will be addressed later. The way to determine the transfer function is well defined: apply a known input signal to the system, and determine $H(\omega)$.

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

(2.29)

In principle one can for $x(t)$ use a signal that is designed contain all the frequency components to be investigated. For instance, if one want to evaluate the transfer function for all integer frequencies from 1 Hz to 120 Hz, such an input signal can simply be generated, i.e. $x(t) = \sum_{n=1}^{120} X_n e^{i2\pi nt}$. The corresponding output, will because of linearity contain the same 120 frequency components, and $H(f)$ can be determined. This method has the advantage that only one time series has to be acquired. The disadvantages of this method is that the different frequency components are sampled with a different number of points per period, and that the pressure modulation system impose constraints to the complexity of the generated signals. The simple and intuitive way is therefore not to superimpose the signals, but to investigate each single harmonic independently, for all $\omega$ of interest. This has the following advantages

- The sampling rate can be optimized for each harmonic, and chosen such that the numerical analysis of each harmonic is identically.
- The analysis of the output signal can be done by means of least squares fitting instead of with a DTFT, which increases the accuracy.
- The signal shapes is the simplest possible, and the signal and its derivatives are continuous, smooth and their maximal values are minimized. This minimizes the constraints imposed by the pressure generating mechanism.
- The quality (linearity, noise) can be assessed directly for each signal.
If the input signal, \( x(t) \), is harmonic, it can be written:

\[
x(t) = \Re \left( X(\omega)e^{i\omega t} \right) = a_x(\omega) \cos(\omega t + \phi_x(\omega))
\]  

(2.30)

where \( A = |X| \) and \( \phi = \arg(X) \) are the amplitude and phase of the signal. The system is linear, why the output signal is of identical form:

\[
y(t) = a_y(\omega) \cos(\omega t + \phi_y(\omega))
\]  

(2.31)

For such an input-output relation the phase shift is

\[
\Delta \phi(\omega) = \phi_y(\omega) - \phi_x(\omega)
\]  

(2.32)

and the gain is

\[
g(\omega) = \frac{a_y(\omega)}{a_x(\omega)} \frac{1}{K} \quad \text{where} \quad K = \lim_{\omega \to 0} \frac{a_y(\omega)}{a_x(\omega)}
\]  

(2.33)

\( K \) is here the static amplification. With these definitions the transfer function \( H(\omega) \) can be written

\[
H(\omega) = Kg(\omega)e^{i\Delta \phi(\omega)}
\]  

(2.34)

The static amplification is not important in terms of unsteady characterization, and is let out when the transfer function is plotted.

### 2.3. Models for PSP response

In this section theoretical models for the frequency response a PSP layer. Two basic models due to [6] will be presented, namely a simple empirical model and a diffusion model. An extension of the diffusion model to account for optical thickness is suggested. The input to the models will be a harmonic pressure variation

\[
P_{\text{in}}(t) = P_0 + P_1 \sin(\omega t)
\]  

(2.35)

where \( P_0 \) is the base pressure, \( P_1 \) is the pressure amplitude and \( f = 2\pi\omega \) is the oscillation frequency. If we non-dimensionalize the pressure with respect to the reference pressure, \( P_{\text{ref}} \), (2.35) becomes

\[
P_{\text{in}}^*(t) = P_0^* + P_1^* \sin(\omega^* t^*)
\]  

(2.36)

For the case where the base pressure is equal to the reference pressure, \( P_0^* = 1 \).
2.3 Models for PSP response

2.3.1. Empirical Models

The simplest approach is to apply an empirical model, without attempting to model the underlying physics of the system. The only assumption needed is that the system is LTI, so it can be meaningfully described with a transfer function, and simply fit a transfer function of guessed form to the experimental data. First-order systems are commonly used as a first approach since their modeling is simple, well-known and the underlying description of a long range of physical systems. For a first order system the transfer function is

\[ H(\omega) = \frac{1}{1 + i\omega\tau} \]  

as derived in appendix App. A.7 on page 103. In [7] the modeling with a first-order system has been conducted for a step-response in the time-domain with some success for both porous PSP and anodized aluminum, but no successful fitting in the frequency domain has been reported. In [6] is reported a reasonable 2-parameter fit to measured frequency response data for a 16-µm PSP sample with a system of the form

\[ H(\omega) = \frac{1 + i\omega\tau_1}{1 + i\omega\tau_2} \]  

The parameters were chosen to fit for the gain, and has considerable deviation for the phase shift. In cases where no simple complex expression for the transfer function can be found to represent the frequency response adequately polynomial fitting can is used to fit the gain and phase shift separately, which is a pure engineering approach.

2.3.2. Diffusion models

In the following a paint layer, as sketched on Fig. 2.4 on the next page, is considered. The layer is considered to be plane and its thickness, \( h \), very small compared to its width and length, such that the diffusion can be considered to be one-dimensional, along the normal to the paint surface. A coordinate \( x \) is used to denote the position in the paint layer between \( x = 0 \) at the model surface, and \( x = h \) at the paint surface. In dimensionless form \( x^* = \frac{x}{h} \).

Relationship between luminiscence and oxygen concentration

Consider the static calibration function (2.13), assuming uniform temperature such that the dependency of \( T^* \) disappears:

\[ I^* = f(P^*) \]  

The intensity \( I^* \) is here the integral intensity measured at the surface of the paint. The intensity per unit depth is defined as \( J = I/h \). In its dimensionless form

\[ J^* = \frac{J}{J_{\text{ref}}} = \frac{I/h}{I_{\text{ref}}/h} = I^* \]  

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At steady-state conditions the concentration of oxygen throughout the layer is constant. From Henry’s law (2.8) the concentration, \( n \), is known at the surface, and so it must have the value \( P^* = P/P_{\text{ref}} = \sigma P/\sigma P_{\text{ref}} = n/n_{\text{ref}} = n^* \) throughout the layer. Altogether, for the steady-state

\[
J^* = f(n^*) \tag{2.41}
\]

As of now, this is an integral equation concerning the intensity leaving the paint surface. If layer is assumed fully transparent such that no attenuation of light happens, (2.41) holds locally according to [6]. If furthermore the diffusion process is much slower than the luminescent lifetime, the system is quasi-static, and the relation is also valid for unsteady phenomena. In [1] the luminescent lifetime of PtTFPP is given to be 50 µs, which is orders of magnitude faster than the time scales investigated here, thus the assumption holds. The oxygen concentration and the intensity is thus now depending on time and depth in the layer:

\[
J^*(x^*, t^*) = f[n^*(x^*, t^*)] \tag{2.42}
\]

To obtain the total luminescent intensity, integrate \( J^* \) across the paint layer:

\[
I^*(t) = \int_0^1 J^*(x^*, t^*)dx^* \tag{2.43}
\]

**Oxygen concentration in paint layer**

The concentration of oxygen in the paint layer, \( n \), as function of \( x \) and \( t \), is now modeled. It is assumed that the transport of oxygen in the paint is diffusive, i.e. that
it is ruled by Fick’s law:

$$\partial^2_x n(x, t) = \frac{1}{D} \partial_t n(x, t)$$  \hspace{1cm} (2.44)

where $D$ is the diffusion coefficient. For porous polymers, Fick’s law can still be valid, if instead of the nominal diffusion constant, the \textit{effective} diffusion constant, $D_{\text{eff}}$ is used. $D_{\text{eff}}$ is depending on several microscopic properties of the diffusion medium, the average length, radius and the fractal dimension of the pores [1]. Determining these quantities is out of the scope for this investigation. The $D_{\text{eff}}$ is simply used as a free parameter for fitting of experimental data, to see if the diffusion model is a adequate description of the paint response at all. The boundary and initial conditions for (2.44) are:

$$n(h, t) = \theta \phi_{O_2} P_{in}(t) \quad \partial_x n(0, t) = 0 \quad \partial_x n(x, 0) = P_{in}(0)$$  \hspace{1cm} (2.45)

This expresses Henry’s law (2.8) at the surface, that there is no diffusion across the model-paint interface, and an initial condition of uniform oxygen concentration throughout the layer. Using the non-dimensional variables

$$I^* = \frac{I}{I_{ref}}, \quad P^* = \frac{P}{P_{ref}}, \quad x^* = \frac{x}{h}, \quad t^* = \frac{tD}{h^2} = \frac{t}{\tau}, \quad \omega^* = \omega \tau$$  \hspace{1cm} (2.46)

and applying the transformation $m^*(x^*, t^*) = n^*(x^*, t^*) - P_{in}^*(t^*)$, the diffusion equation (2.44) and the boundary and initial conditions (2.45) become

$$\partial^2_x m^*(x^*, t^*) = \partial_t m^*(x^*, t^*) + d_t P_{in}(t^*)$$  \hspace{1cm} (2.47)

and

$$m^*(1, t^*) = 0, \quad \partial_x m^*(0, t^*) = 0, \quad \partial_x m^*(x^*, 0) = 0, \hspace{1cm} (2.48)$$

Using modal analysis it is demonstrated in [6] that this equation has a separable solution of the form

$$m^*(x^*, t^*) = \sum_{k=1}^{\infty} \alpha_k(t^*) \Psi_k(x^*)$$  \hspace{1cm} (2.49)

where

$$\Psi_k(x^*) = \cos(\lambda_k x^*) \quad \text{and} \quad d_t \alpha_k(t^*) = -\lambda_k^2 \alpha_k(t^*) - \frac{2 \sin(\lambda_k) d_t P_{in}^*(t^*)}{\lambda_k}$$  \hspace{1cm} (2.50)

and where $\lambda_k = \pi (2k - 1)/2$. Therefore the solution to for the oxygen concentration in the polymer is given by

$$n^*(x^*, t^*) = \left( \ldots, \cos(\lambda_k x^*), \ldots \right) \left( \begin{array}{c} \vdots \\ \alpha_k(t^*) \\ \vdots \end{array} \right) + P_{in}^*(t^*)$$  \hspace{1cm} (2.51)
as given in [6]. To solve this for the present pressure variation, (2.36) is inserted, and solving the resulting ordinary differential equation for $\alpha_k(t^*)$ yields

$$\alpha_k(t^*) = \frac{2 \cos(\pi k) \omega^* P_1^* \left[ \lambda_k^2 \cos(\omega^* t^*) + \omega^* \sin(\omega^* t^*) \right]}{\lambda_k^4 + \omega^*}$$

(2.52)

and together with the pressure boundary conditions

$$P_m^*(t^*) = 1 + P_1^* \sin(\omega^* t^*) \quad \text{and} \quad d_e^* P_m^*(t^*) = P_1^* \omega^* \cos(\omega^* t^*)$$

(2.53)

the oxygen concentration profile can be evaluated to arbitrary precision by evaluating a sufficient number of modes of (2.51), and by means of (2.42), the luminescent intensity is obtained by numerical integration of (2.43),

**Optical Thickness**

In order to account for the internal light attenuation, a simple model is here proposed. A modification of (2.42) is made such that

$$J^*(x^*, t^*) = \eta(x^*) f[n^*(x^*, t^*)]$$

(2.54)

which expresses that the actual intensity contribution from the depth of $x^*$ is not only depending on the concentration, but also explicitly depending on $x^*$. For $f$ to still be the measured steady calibration function, it must hold that $\int_0^1 \eta(x^*) dx^* = 1$, in which case the static calibration (2.41) is still valid. In other words $\eta(x^*)$ is a weight function applied to the concentration profile to get the actual intensity contribution profile.

The penetration of light into solid matter can in general be described by an exponential model [8]. If a surface is illuminated with intensity $I_{ex}$, the intensity penetrating into the position $x$ in the layer. The thickness of the layer the light have to pass through is $h - x$, therefore

$$I(x) = I_{ex} e^{-\frac{h-x}{\sigma}}$$

(2.55)

where $\sigma$ is the optical thickness which depends on the medium and the wavelength of the light. For luminescent response this has to be applied for both the incoming and outgoing light. Consider the situation where absorption takes place at $\lambda_1$ and the emission at $\lambda_2$. The detected luminescent intensity from the paint, $J_{lum}$ is then

$$J_{lum}(x) = J_{ex} \xi^e \frac{h-x}{\sigma_1} \cdot e^{-\frac{h-x}{\sigma_2}}$$

(2.56)

where $I_{ex}$ is the excitation intensity, and $I_{lum}$ the thickness-specific luminescent intensity both measured outside the paint layer. $\xi$ denotes the local conversion efficiency of impacting light to luminescent light, which is a function of various quantities such as oxygen concentration, calibration function, luminophore desity etc.
It is not, however, function of $x^*$ explicitly. By redefining the optical thickness for this specific application:

$$\frac{1}{\sigma} = \frac{1}{\sigma(\lambda_1)} + \frac{1}{\sigma(\lambda_2)} \quad \text{and} \quad x^* = \frac{x}{h}, \quad \sigma^* = \frac{\sigma}{h}$$

(2.57)

into (2.56)

$$J_{\text{hum}}(x) = J_{\text{ex}}\xi e^{-\frac{x^*}{\sigma^*}}$$

(2.58)

For the idealized case of an unattenuated signal, $\sigma^* \to \infty$ and $J_{\text{hum}}(x) = J_{\text{ex}}\xi$. The ratio between local intensity contribution for the attenuated and the unattenuated light signal respectively, is thus $e^{-\frac{x^*}{\sigma^*}}$. Therefore

$$\eta(x^*) = e^{-\frac{x^*}{\sigma^*}} \int_0^1 e^{-\frac{x^*}{\sigma^*}} dx^* = k e^{-\frac{x^*}{\sigma^*}} \quad \text{where} \quad k = \left[ \sigma^* \left( 1 - e^{-\frac{1}{\sigma^*}} \right) \right]^{-1}$$

(2.59)

$\eta(x^*)$ denotes the ratio of a real to an unattenuated signal, normalized for compliance with the steady calibration function. Physically, if $\sigma^* \gg 1$, the light attenuation can be disregarded, whereas for $\sigma^* \ll 1$ only a small part of the layer actually contributes to the luminescent intensity.

**The linear calibration function**

Since (2.51) is linear with respect to the input pressure, if the calibration function is linear, then also the integrated intensity, (2.43), will be so. In [6] a linear PSP model is investigated, namely

$$f(P^*) = I^* = a_0 + a_1 P^*$$

(2.60)

This is a linearized model, which is only valid locally for small pressure amplitudes, consist of the two first terms of a Taylor expansion of the real calibration function. If the system is properly described by this model, the system can be regarded linear, whereas the Stern-Volmer relation or a higher order calibration functions, is nonlinear, and in principle renders the whole analysis method meaningless. That the nonlinearities for the present paint are small enough to neglect will be shown in the following chapter. The linear calibration combined with the diffusion model has the transfer function of

$$H(\omega^*) = \frac{\tanh(\sqrt{i\omega^*})}{\sqrt{i\omega^*}}$$

(2.61)

according to [6]. A graphical representation of this transfer function as well as that of the systems of function as well of the models of Sec. 2.3.1 on page 13 are seen in App. A.8 on page 104.
2.4. Experimental Unsteady PSP Characterization

This section contains a discussion of the advantages and disadvantages of characterization in the time and frequency domain respectively, combined with a brief review of unsteady characterization techniques previously used for PSP.

2.4.1. Time-domain characterization

The majority of the results of unsteady PSP investigations so far have been limited to the time domain. For testing of new PSP formulations, to get an order of magnitude of the dynamic response, such investigations are sufficient.

The principal way to do a time-domain characterization is to measure the impulse response, \( h(t) \) in (2.27). With this approach, the result could be transformed directly to the transfer function. The reason this is not done is that, as explained in appendix App. A.1 on page 98 the impulse response is mathematically described as a Dirac delta function, which can principally not be reproduced in practice. In fact it is hard to generate a even reasonable approximation to such a pressure signal in practice. For other types of signals (e.g. electronic or light signals) it is possible to create a short well-defined pulse (as e.g. in [9]), whereas for pressure signals this would require a very sophisticated setup. Even if a approximation to the signal can be generated, extreme care still has to be exercised in the analysis, since the concrete shape and duration can have significant influence on its spectral content. For example will, say a square pressure pulse of 100\( \mu \)s duration have a very outspoken frequency component at 5kHz, and its spectrum will be very different if it was a triangle pulse instead. Especially the excitation of system eigenfrequencies is very dependent on the actual spectral content.

In practice, instead of an impulse response, a step response is typically investigated. For this a shock tube or a similar apparatus is typically used. With this type of equipment a membrane separating two chambers with different pressures is burst, causing a pressure equalization. This type of setup can give a pressure change in pressure level happening in the order of 100\( \mu \)s [10]. Investigations of this type were for instance made for fast-responding PSP’s [7, 11] with a shock tube, and for polymer-based PSP’s with vacuum systems [10, 12]. For time-domain characterization and relative comparison of PSP dynamic performance these methods are suitable, however the results are limited to interpretation in the time domain. In [7, 12] the results were fitted with the step response of a first order system (A.37) , with reasonable results. First-order system fitting is employed, not so much because the system is really first-order, as for having obtaining a simple one-parameter model in which the characteristic response time, \( \tau \), has a well-defined analytical interpretation. There are two factors which furthermore inhibits the interpretation of these results in the frequency domain. Firstly, shock tubes and similar equipment produce pressure steps that are much more drastic than desired. In [11] the pressure step is approximately 30kPa, whereas in [10] it is three times as much. For these pressure
steps, the non-linearity of the calibration function becomes important, which invalidates the assumption of an LTI system. Also, the amplitude of the pressure changes in itself can influence the time response, which is observed in [13]. Another problem is that the nature of these investigations make a high number of accurate reproduction measurements difficult and tedious. This makes noise-reduction averaging in the time-domain impractical, and since a high accuracy is required for the time response in order to do a meaningful frequency analysis.

2.4.2. Frequency-domain characterization

There are great advantages by characterizing PSP in the frequency domain. Applying an input signal with well-known spectral content instead of a short impulse, sets requirements for the experimental setup, but is worth the effort. A direct frequency-response experiment on PSP was published by Carroll, Winslow et. al. [13]. This is the first and only paper known to the author where a direct-modulation approach to PSP response has been applied. The experiments were carried out for conventional PSP by low frequencies \( f < 2\text{Hz} \). The pressure modulation mechanism was an analog valve, which put imposed natural limits upon the quality and accessible frequency range of the signal. The transfer function was found by using a 0.1 Hz sawtooth signal for modulation, and through DFT analysis obtaining the first 20 harmonics \( f = [0.1, 0.2, \ldots, 2.0] \text{Hz} \). They also report an observed sensitivity of the transfer function to the pressure amplitude. These measurements are reported in [13] and [14].
3. Hardware and Experimental Setup

In this section the investigated PSP will be described, and the experimental setup its unsteady characterization

3.1. iPSP paint

Various PSP formulations are under continuous development in a cooperation between DLR Göttingen and the Chemical institute of the University of Hohenheim, among these the family of unsteady paints investigated in this work. The main formulation is termed iPSP, to emphasize the unsteadiness of the PSP. The iPSP is based on the luminophore Platinum Tetra(pentafluorophenyl)porphine (PtTFPP) whose absorption peak lies at 388 nm [15], and emits at 650 nm. PtTFPP has a luminescent lifetime of $50\,\mu s$ [1], and can thus principally be used for measurement of pressure oscillations up to kHz-level. The containing polymer was a fluoroacrylic polymer, Fluoro/Isopropyl/Butyl (FIB) [15]. The combination of PtTFPP and FIB is renowned to have low temperature sensitivity [1, 15], and being very close to "ideal", meaning that the temperature and pressure sensitivity are essentially decoupled and the mixed-term coefficients in (2.14) are very small. The iPSP is a binary PSP, with the reference component, NP-108-62, being a phosphorescence pigment based on an Europium complex, which emits at 518 nm. The emission spectrum of the iPSP is seen on Fig. 3.1 on the next page, measured for three different temperatures, and two different pressures. Both pressure and temperature dependence of the intensity is seen for the pressure component, whereas the reference component is virtually unaffected by this.

To provide porosity, rutile TiO$_2$ particles were added to the paint. Apart from possessing the primary desirable properties, high pressure sensitivity and luminescent output and low temperature sensitivity and response time, the iPSP formulation had to meet a range of additional requirements, in order to be applicable for industrial measurements. It should be applicable with a spray gun, be adhesive, and have a reasonable drying time. In its dried state it should be durable enough to withstand the aerodynamic forces. The surface should for most applications be aerodynamically smooth, at best it should be polishable, which requires that it is sufficiently hard and hard and

\[1\] The $i$ in iPSP stands for "instationä"r", the German term for unsteady.
3.1 iPSP paint

Reference component

Pressure component

\[ p = 100 \text{ kPa} \]

\[ p = 10 \text{ kPa} \]

\[ \lambda / \text{nm} \]

\[ I / \text{au} \]

\[ T = 20^\circ \text{C} \]

\[ T = 30^\circ \text{C} \]

\[ T = 40^\circ \text{C} \]

**Figure 3.1.** Emission spectrum for the iPSP at different pressures and temperatures
3.2 Unsteady Calibration Setup

The unsteady calibration setup realizes a way of which the PSP probes can be exposed to spatially invariant pressures modulated with a high degree of accuracy and over a broad range of frequencies from the quasi-static (for this application in the order of 0.1 Hz) up to kHz-level, and for amplitudes of up to 3 kPa. The setup heart of the setup is pictured on Fig. 3.2. It consists of a pressure chamber (A), a 50mm × 50mm × 5mm chamber, in which the probe is placed. The pressure chamber has a glass front, for optical access to the probe. The chamber is open in one side, which is connected with a cylindrical chamber (B), whose end wall is made up by a membrane. The position of the membrane determines the combined volume of the membrane and calibration chamber. The position of the membrane is controlled through a thin lever (D) (to eliminate shear forces on the membrane) by a mechanical vibration exciter, a so-called shaker (C) (Tira). The philosophy of this setup is that a pressure modulated by the forced volume change of a closed system is very accurate and very spatially uniform, since no valve with possible leakage, flow restrictions, local high-speed flows and

not sticky. The iPSP formulation fulfills these requirements reasonably, and was used for the wind tunnel test described in chapter 6. In the final formulation the mass ratio of the FIB/NP-108-62/TiO₂ was 3/9/7. For unsteady calibration the paint was sprayed onto 30mm × 30mm aluminum plates.
limited control is involved. A normal loud speaker system could not fulfill the requirements of high force, high accuracy and broad range of operating frequencies for the pressure modulation. While actually being designed as an exciter for to structural dynamics applications, the shaker possessed exactly these virtues. The shaker piston displacement is ruled by a DC current, which is supplied by a power supply (Tira). The system can deliver high-quality function shapes in the range from below 0.1 Hz up to the kHz domain, however in order keep the displacement, velocity and accelerations within the rated values for the shaker, the pressure amplitude must be limited at high frequencies. Between 0-60 Hz the pressure amplitude used for experiments was 3 kPa, while between 60 and 120 Hz this was varied linearly down to 1 kPa. A detailed exploded view of the pressure chamber is seen on Fig. 3.2.


When the pressure chamber is closed the probe (A) and the front window (B) is mounted to the chamber. The pressure chamber part of the setup is equipped with four Endevco 8510B-5 piezoresisitive pressure transducers (C) in the following referred to as the reference pressure sensors. These sensors have a very low response time, and can for the present pressure oscillation frequencies be regarded as immediately responding.² The side wall with the slit (D) leads to the membrane chamber. The temperature in the calibration chamber is kept constant by a Peltier element (E). The

²The working principle of this type of sensor is that a diaphragm is subjected to the pressure. The diaphragm incorporates piezoresisitive elements, making it possible to deduce the mechanical deformation from changes in the resistance, and from that to compute the pressure on the diaphragm. By
calibration chamber itself is made from a copper alloy for quick thermal stabilization. The base pressure in the calibration chamber is ruled by a pressure controller (Paroscientific) connected through a plastic tube. During measurements this connection to the chamber is closed with a magnetic valve on the calibration chamber side (F), to eliminate pressure reflections in the tube. The whole setup is mounted with hard rubber blocks on a big aluminum plate, to reduce vibrations. The unsteady calibration setup is protected by patent DE 10 2005 047.623.6-52. On Fig. 3.2 a sketch of the complete measurement system is seen.

Figure 3.4.: The measurement system, including light source, detector and control

Apart from the already mentioned components, the excitation light source and the light detector is shown. Almost all aspects of the setup can be controlled by means of the measurement computer. The settings for the respective controllers of the systems are set, in particular the pressure modulation information which is sent to a function generator (Yokogawa FG320), which then generates an analog signal as input for the power amplifier, ruling the current to the shaker, and hence the displacement of the membrane. Acquisition of data from the reference pressure sensors and from the means of modern microfabrication these sensors are given very small dimensions, correspondingly high resonance frequencies and thus unsurpassed dynamic characteristics. The diaphragm of the present sensor has according to the manufacturer its first resonance frequency at 85 kHz. For a brief review of the principles and the state-of-the-art of commercial micromechanical pressure transducers see [16].
light detector is done by means a signal recorder (Yokogawa ScopeCorder DL750) which performs 16-bit A/D-conversion and transfers the data over ethernet to the measurement computer. The reference pressure sensors are individually and statically calibrated by means of the pressure controller. The linearity of the sensors is excellent, and a three-point calibration of the sensors is sufficient. A second order calibration function is used, and it is ascertained that the second order term is negligible.

3.3. Light Detectors And Optics

In front of the chamber the excitation light source and the light detector is placed. On Fig. 3.2 on page 23 mounting rails for optics are shown. Actually these were not used in the final setup, due to resonance between the table and the shaker at approximately 25 Hz. Instead the optical components were mounted on a rail, entirely decoupled from the table. As excitation light source an array of 26 high power light emitting diodes (LEDs) were used (see Fig. 3.5a). It is a custom-built by the company RAPP OptoElectronics GmbH, and delivers a highly stable continuous light at a wavelength of 405 nm. The light detector is the most critical component in the setup, as the requirements are many and high:

- **Good temporal resolution** First and foremost, since the measurements are unsteady, the light detector needs to have very high temporal resolution. For example to resolve, say a 100 Hz signal with 512 points per period, the signal must be sampled with 51.2 kHz. On the other hand the spatial resolution, which is after all the boast of the PSP method, is not needed for this investigation, where the light signal from the whole probe is integrated.

- **High sensitivity** The intensity signal from the paint is quite low, and the high temporal resolution makes it impossible to increase the intensity by increasing exposure time, as can be done by steady experiments. The light detector thus needs a high sensitivity around $\lambda = 650\text{nm}$, and large amplification.

- **Frequency response and stability** The frequency response of the light detector should ideally be negligible, and at least be reproducible such that a correction can be made. Also the light detector must yield a stable signal over time and not exhibit significant hysteresis or drift.

Different types and brands of light detectors were tested (Electron multiplication CCD (EMCCD), Photodiode (PD), Avalanche Photo diode (APD) and 2 different photomultiplier tubes (PMT)). It was found that only the PMT’s had sufficient sensitivity at the desired wavelength. The two different PMTs were used for simultaneous measurements, namely a Hamamatsu R5863P (PMT 1) and a Hamamatsu R7400U-20 (PMT 2). This was done since the PMT’s showed complementary performance with regards to stability at high and low frequency, and to
cross-verify the measurements. Unless otherwise stated, the results in the following are for the PMT 2.

![LED array and PMT 2 with optics](image)

**Figure 3.5.** a) The LED array with filter used for excitation of the PtTFPP luminophore.  
b) Hamamatsu R7400U-20 Photomultiplier tube (PMT 2) with c-mount objective and optical filter

Before the PMT an objective and an optical filter has been mounted. The objective have been chosen, such that the field of view at the viewing distance of the PMT (≈ 0.5 m) coincides with the paint probe area. The distance between objective and PMT photocathode has been chosen such that the image size from a 30×30 mm object at 0.5 m distance with fully open aperture, fits approximately with the photocathode size. This is done to assure an optimal illumination of the photocathode with regards light amount and homogeneity. The chosen objective for PMT 2 is a 50mm c-mount objective with continuous aperture. In front of the objective an optical bandpass filter (Andover Corporation 650FS80-50) is inserted. The filter bandwidth is 80 nm centered around 650 nm, which covers the emission peak of Fig. 3.1 on page 21 without collecting light from the reference luminescence or the excitation light.
4. Setup Characterization

This chapter deals with the aspects of the experimental setup for the paint characterization.

4.1. Measurement Procedure

In this section the measurement procedure for obtaining a transfer function is described in general terms. For instance generic terms as "input signal", "output signal", "illumination" and "light detection" will be used as collective terms. This is due to the fact that the experiments used for characterization of the setup itself and for characterization of the paint, are principally identical, as will be seen. On Fig. 4.1 an overview of the measurement system is seen.

![Figure 4.1: An overview of the different configurations of the measurement system. Illumination of the light sensor is done by either by the PSP probe (B), with the input signal being the pressure from the reference pressure sensor, or by direct illumination with an LED (A), in which case the input measured as the voltage over $R_2$. For light detection is either used a Photo diode (C) or PMT (D). For standard PSP measurements configuration B-D is used, while A is used in combination with C or D for characterization measurements.](image)

In the standard configuration for unsteady PSP measurements the measured intensity signal comes from PSP (B) and is detected by a photomultiplier (D). The input signal
is the reference pressure measured by the upper left *Endevco* sensor. For the characterization of the light sensors, alternative illumination and detection is used. The generic measurement procedure is as follows: The aperture is adjusted such that the desired output signal level is obtained. The light source and light detectors are turned on, modulation of the illumination initiated, and after an optional stabilizing period for the system, measurements are acquired. Successive time series measurements for ascending pressure modulation frequencies. Three ranges for modulation frequency ranges are used, namely $f = 0.1, 0.2, \ldots, 2.0$ Hz (for slow paints), $f = 1.2, 120$ Hz (for most measurements), and $f = 2.5, 5.0, \ldots, 1000$ (for setup characterization). These will be referred to as the "low", the "intermediate", and the "high" frequency range. For each frequency a time series is acquired, containing a fixed number of samples, $N_s$. The sampling frequency, $f_s$, is chosen from the pressure modulation frequency to be $f_s = N_{pp} f$, where $N_{pp}$ is the number of points per period which is held fixed for all measurements. The choices for these parameters presented in Tab. 4.1 are used for all measurements.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation frequency</td>
<td>$f$</td>
<td>[0.1 - 1000] Hz</td>
<td></td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>$f_s$</td>
<td>256 · $f$ Hz</td>
<td></td>
</tr>
<tr>
<td>Samples per time series</td>
<td>$N_s$</td>
<td>25000</td>
<td>-</td>
</tr>
<tr>
<td>Periods per time series</td>
<td>$N_p$</td>
<td>97</td>
<td>-</td>
</tr>
<tr>
<td>Samples per period</td>
<td>$N_{pp}$</td>
<td>256</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.1.: Sampling parameters for setup and unsteady PSP characterization

### 4.1.1. Data reduction to determine frequency response

The choice of the parameters in Tab. 4.1 are chosen from the following considerations: That $N_s$ and $N_{pp}$ are constant has obvious advantages for the simplicity of the data acquisition, storage and reduction. That $N_{pp}$ is an integer allows for time-domain averaging and that the Discrete-Time Fourier Transformation (DTFT) of the signal reduces to a Discrete Fourier Transformation (DFT) (See appendix App. A.4 on page 100). That $N_{pp}$ is furthermore a power of two, enables exact calculation of the DFT by means of the FFT algorithm\(^1\). The output signal from the PMT has quite a low signal-to-noise ratio (SNR), which is the reason why time-domain averaging is

---

\(^1\) A DFT, in comparison to a general DTFT, eliminates leakage. Sampling at a frequency which is a multiple of the modulation frequency removes all the intermediate frequency components caused by the finite length of the time series, since they are all evaluated at their zero crossings. Physically this corresponds to assuming that the sampled time series repeats itself on and on to infinity, thus throwing away all information that has to do with the finite sampling time. This is exactly what one would want to in the present case. The FFT algorithm can be used for a signal consisting of $2^N$ points, and choosing $N_{pp} = 2^N$ makes it possible to use it without losing the advantage (by having to do zero padding) obtained by the first simplification.
employed. It is eventually the low SNR which sets the limit for how high frequencies can be analyzed, as the SNR becomes smaller from 60 Hz and onwards, on grounds of the reduced input pressure amplitude. For a uniformly sampled signal the time-domain average is given as

$$\bar{x}_k = \frac{1}{N} \sum_{j=0}^{N-1} x_{jN+k} \quad \text{for} \quad k \in \{0, 1, 2 \ldots N-1\}$$

(4.1)

where $k$ is the number of evaluation points of the time-domain average, and $N$ the number of averaging values. In the present case, the ensemble averaging are made over two whole periods, i.e. $k = 2N_{pp} = 512$ and $N = \text{mod}(\frac{N_{pp}}{k}) = 48$. Then the input and output signals are fitted by sine signals of the form

$$x(t) = c_x + a_x \sin(\omega t + \phi_x) \quad \text{and} \quad y(t) = c_y + a_y \sin(\omega t + \phi_y)$$

(4.2)

This is done by an iterative least squares (LS) fitting method developed in [17] and described in more detail in appendix App. B on page 106. This method has some advantages over using a DFT, since a DFT frequency spectrum is reduced to multiples of a fundamental frequency. For the iterative LS method this resolution is in the order of machine precision, and allows for checking input and output frequency with high precision. On Fig. 4.2 on the following page an example of the time raw signal from the iPSP measured with PMT 2 is seen (configuration B-D on Fig. 4.1 on page 27). The input pressure signal (green) is seen to be virtually perfectly sinusoidal, while the PMT output (red) is quite noisy. The blue graph shows the time-domain average of the PMT signal, and the black lines show the LS fits to the input and output signals respectively. It is seen that the time-domain averaging reduces the noise significantly\(^2\), and that the sinusoidal fitting represents both input and output signals beautifully.

This shows that the underlying system is close to linear, since nonlinearities would distort the sinusoidal shape of the output signal, as discussed in the next section. Since the linear relation between the input and output (2.60) has a negative coefficient, there is a sign difference in between the input and output on Fig. 4.2 on the following page. In the following this sign will be inverted, such that signals as on Fig. 4.2 on the next page are considered to be in phase. With this in mind, it is seen that here, for $f = 1$ Hz, the phase shift is very close to zero.

**Determination of gain and phase shift**

From the fitted input and output signal at a given frequency $f$, the value of the transfer function for this frequency can be determined. From the sinusoidal fits the phase of the two signals are determined, and the phase shift determined as the difference (2.32). This measurement is direct, whereas the gain can only be determined by normalization,\(^2\)

\(^2\)To be precise: If the noise is random it is decreased by a factor of $\sqrt{N}$ where $N$ comes from (4.1), in the present case a factor of approximately 7.
Figure 4.2: Example on iPSP signal measured by PMT 2. Signals have been offset zero and normalized to unit amplitude.
refer to its definition (2.33). In this equation, the limit value for the amplitude ratio as the frequency tends to zero, must be approximated by the value of the amplitude ratio at the lowest available frequency. The gain, $g(\omega)$, is therefore approximated:

$$g(\omega) = \lim_{\omega' \to 0} \frac{a_y(\omega)}{a_x(\omega)} \approx \frac{a_y(\omega')}{a_x(\omega')}$$  \text{ where } \omega' \text{ is small} \quad (4.3)$$

For the same paint sample as that of Fig. 4.2 on the preceding page, the fitted signals are shown for $f = 1$ Hz and $f = 100$ Hz on Fig. 4.3. The time is normalized with respect to the period, $T$, and the normalization of the output signal is with respect to the input signal and the amplitude ratio of the signal at 1 Hz, in accordance with (4.3), with $f = 1$ Hz as the lowest frequency. Also the inverted input pressure is plotted. It is seen from Fig. 4.3 that the intensity at $f = 1$ Hz has virtually no phase shift with respect to the (inverted) input signal. In contrast, the signal at 100 Hz lags considerably in time compared to the input signal, and the amplitude is reduced to one fourth of the value at 1 Hz. The gain at 100 Hz is due to the normalization simply the amplitude of the 100 Hz - time signal, whereas the phase shift, $\Delta \phi = -\frac{2\pi \Delta t}{T}$ in terms of the time difference between output and input $\Delta t$.
Whether the a frequency is small enough for (4.3) to have converged can be assessed from the phase shift for that frequency, which must also tend to zero in the limit. If the phase shift is close to zero, it can be assumed that also the gain is so. Since the real measurements of gain are subject to noise, the normalization is not done with respect to the value in the first point. Instead a polynomial of $k^{th}$ order is fitted to the first $N$ points in the measurement series, and evaluated at the lowest frequency. In this way the normalization is made less sensitive to the noise, or to short transients for the lowest frequencies. If at the lowest measured frequency, the phase shift is not zero, a detailed investigation at low frequencies is done. The data from the two transfer functions are then merged. This is no problem for the phase shift measurements since they are absolute, but the gain must be re-normalized. If measurements at $f = 0.1, 0.2, ..., 2.0 \text{ Hz}$ and $f = 1, 2, ..., 120 \text{ Hz}$ are available, the corresponding gain data are merged at a common point, in this case at $f = 1 \text{ Hz}$. Also in this case, a polynomial of $k^{th}$ order is passed through the last $N$ points of the low-frequency gain series and another through the first $N$ points of the high-frequency gain series. The polynomials are evaluated at the joining point, and the high-frequency gain data are then re-scaled, so the fitting polynomials share value in the joining point. For the present investigations $N = 4$ and $k = 10$ was found to provide smoothly looking junction of the gain data.

### 4.2. Verification of LTI system properties

To justify the analysis method it is in this section shown that the measurement system, including the paint, acts linearly for the range of pressure amplitudes used for the measurements.

#### 4.2.1. Steady Calibration

Figure 4.4a shows the steady calibration measured for the iPSP at three different temperatures. As is customary, the ordinate is $1/I^*$, since Stern-Volmer relations appear linear in such a plot. The steady calibration has been conducted in a setup similar to the present, but with steady pressure. The applied fit on is of the form of (2.14), which is seen to be an accurate representation of the measurements. In the following it will be investigated, whether a linearizing the steady calibration function provides a sufficiently accurate description for the pressures locally. The linear, Stern-Volmer and second-order calibration function are respectively

\[
\begin{align*}
 f_0(P^*) &= I^* = a_0 + b_0 P^* & \text{Linear} \\
 f_1(P^*) &= I^* = \frac{1}{a_1 + b_1 P^*} & \text{Stern-Volmer} \\
 f_2(P^*) &= I^* = \frac{1}{a_2 + b_2 P^* + c_2 P^{*2}} & \text{Second order (4.4)}
\end{align*}
\]
4.2 Verification of LTI system properties

Setup Characterization

\[ I = \frac{300}{I_P} \text{kPa} \]

- \( I = 30^\circ C \) (incl. cal. ex)
- \( I = 40^\circ C \) (incl. cal. ex)

(a) Measured calibration

(b) Three calibration models

Figure 4.4: (a) Steady calibration of iPSP for three different temperatures, fitted with . (b) The three calibration functions. The second order model is fitted to all of the data. The two other models are fitted to the value and slope of the second for \( P^* = 1 \).

These three expressions fitted to the \( T = 30^\circ C \) measurement of Fig. 4.4a, are seen on Fig. 4.4b (with \( I^*(P^*) \) as ordinate). The second order model it that of Fig. 4.4 is fitted to the measured data by means of (2.14), whereas the coefficients of the two others are analytically derived to match value and slope of the second-order model at \( P^* = 1 \). Hence, they are only to be considered local approximations. It is interesting to see how good the linearized model performs compared to the second-order, in terms in pure steady approximation. Depicted on Fig. 4.5a is the output of a sinusoidal input signal, \( P_{in}^* = P_{0}^* + P_{1}^* \sin(\omega^* t^*) \) consistent with (2.36), fed to both models at atmospheric pressure. The linear output, represented by the red curve, is sinusoidal regardless of \( P_{0}^* \) and \( P_{1}^* \). For the second order model this on holds for \( P_{1}^* \to 0 \), while for increasing amplitude the nonlinearity becomes evident, especially toward the low-pressure side. From Fig. 4.5 on the following page it is seen that for \( P_{1}^* = 0.1 \), the signal is still close to sinusoidal. Since the highest pressure amplitudes in the pressure chamber are in the order of \( P_{1}^* = 0.03 \), the linearization of the calibration function is clearly valid. A general overview of the quality of the fit is obtained by comparing the the oscillation part of the second-order and the linear model. Taking relative deviation between these two, \( \epsilon \), and evaluating it for the lowest pressure within the oscillation,
4.2 Verification of LTI system properties

Setup Characterization

\[ \frac{r(t) - I_0}{P^*_1} \]
\[ t/T \]

\[ P^*_1 \rightarrow 0 \]
\[ P^*_1 = 0.1 \]
\[ P^*_1 = 0.25 \]
\[ P^*_1 = 0.5 \]

(a) Intensity response

(b) Max. relative error for \( P^*_0 \) and \( P^*_1 \)

Figure 4.5.: (a) Intensity response to harmonic pressure signal, for different pressure amplitudes at \( P^*_0 = 1 \). The curves have been offset and scaled with \( f(P^*_0) \) and \( P^*_1 \), for making shape comparison possible. (b) Contour showing the largest relative intensity error (with respect to the second-order model) of the linearization.
Figure 4.6: Simulated frequency response for the three calibration functions combined with the PSP diffusion model. $P_0^* = 1$ and $P_1^* = 0.5$

\[
\epsilon_{\text{max}}(P_0^*, P_1^*) = \frac{[f_2(P_0^* - P_1^*) - f_2(P_0^* - P_1^*)] - [f_0(P_0^* - P_1^*) - f_0(P_0^*)]}{[f_2(P_0^* - P_1^*) - f_2(P_0^*)]}
\]

\[
= \frac{P_1^*[3c_2^2P_0^* + (-2c_2^2P_1^* + 3b_2c_2)P_0^* + b_2^2 - c_2a_2 - b_2c_2P_1^*]}{(a_2 + b_2P_0^* + c_2P_0^*)^2(b_2 + 2c_2P_0^* - c_2^2P_1^*)}
\]

where the last equation has appeared by evaluating $f_0$ as the two first terms of the Taylor series of $f_2$ around $P_0^*$. A contour plot of the size of $\epsilon_{\text{max}}$ is seen on Fig. 4.5b. It is seen that at the operating conditions for the calibration chamber, $P_0^* = 1$ and $P_1^* = 0.03$, the maximal error is a few percent, which is acceptable, since the typical error is significantly less than the maximal. Even when the error is significant, it does not have a devastating effect on the analysis, even though it is principally wrong to conduct it. This is exemplified by Fig. 4.2.1, where the three calibration models have been used to compute the frequency response numerically. The output for sinusoidal pressure input has been simulated and fitted by the procedure described in Sec. 4.1.1 on page 28. For the shown figure $P_0^* = 1$ and $P_1^* = 0.5$, corresponding to the green curve on Fig. 4.5a for the second-order calibration. It is seen that regardless of the non-sinusoidal output signals of the second-order model, the shape of the resulting transfer function remains virtually unaffected.

It is concluded that the paint itself is properly described as a LTI system for pressure amplitudes of less than 3 kPa at a base pressure 100 kPa, and that considerably higher amplitudes can potentially be meaningfully analyzed.
4.2.2. LTI properties of system

Any general periodic and bounded signal can be represented by a Fourier series, i.e. a number of harmonic signal components and an offset. If linearity and time-independence can be demonstrated for both the amplitude of harmonic signals and for steady measurements, LTI properties have been demonstrated. The following experiments are all conducted with the original iPSP sample. By keeping all other than the pressure amplitude held constant, the amplitude linearity is investigated, and the results shown for $f = 60$ Hz at Fig. 4.7, where the PMT signal amplitudes are shown as function of pressure amplitude measured by the reference sensor. It is seen that the calibration system responds virtually perfectly linear to amplitude changes. Furthermore, since repeated measurements were made in the opposite chronological order, the system is time-independent with regards to this process.

![Figure 4.7: Amplitude linearity for a 60 Hz sinusoidal pressure signal](image)

A steady calibration of the iPSP around atmospheric pressure were made in the unsteady setup. The pressure were set to a static value, and time series of 1 seconds length acquired for the steady pressure and intensity values. The mean values of the time series were used as values and the standard deviations as error estimates. The absolute PMT signal given as function of $P - P_0$. 

36
The result is seen on Fig. 4.8.

Figure 4.8.: Offset Linearity for PMT 2. The measurements have been acquired starting from at $P = P_0$, then ascending, then descending and then ascending back to $P = P_0$. The error are based on standard deviation of the 1-second time series.

It is clearly seen that the relationship is linear, but also that there seems to be some systematic drift, i.e. time dependence over the little more than 20 minutes the measurements lasted. This is probably also be related to the fact that the PMT needs some stabilization time, before it delivers a relatively stable output. This is illustrated by Fig. 4.9 on the next page, which shows the result for the gain of the two PMT’s for simultaneous measurements at $f=100$ Hz without changing of any parameters. The measurements have been started immediately after turning on the PMT’s and the LED, and acquired successively without break. Assuming that the signals are fully stabilized at the last measurement, the gain has been renormalized to 1 at this value.

Two conclusions are drawn from the figure. The PMT’s both exhibit significant transient effects when exposed to a change in signal level, as seen from the first couple of minutes. This is interpreted as change in noise level due to thermal inequilibrium within the PMT, caused by the change in cathode current. Also the signals are seen to correlate significantly, for instance at the local peak at the tenth minute. This tells that rather than being random or due to sensor specific drift, these fluctuations reflect real changes in the intensity. Thus Fig. 4.9 on the following page also provides a measure for the stability of the excitation light source. The transient must be avoided, hence before starting data acquisition, the measurement system is allowed to run with everything else than data acquisition active, for 15 minutes, in which case the
transients are eliminated and the fluctuations in the measured intensity are less than one percent.

### 4.3. Transfer function for light sensors

In order to determine the frequency response of the paint itself, it has to be known for the whole measurement chain as well. If any frequency-dependence in the gain or phase shift is present here, it must either be small enough to neglect, or reproducible enough to correct for. Since both the paint itself and the total system (measurement system incl. paint) are linear systems, the frequency response from the total system can be described as

$$H_{\text{system}}(\omega) = H_{\text{paint}}(\omega)H_{\text{sensor}}(\omega)$$

where $H_{\text{sensor}}(\omega)$ is the transfer function of the paint itself, while $H_{\text{sensor}}(\omega)$ is the transfer function caused by the rest of the sensor system. This includes mainly the frequency response of the pressure and light sensors and the electrical circuit, but also any discrepancy between measured/assumed values and real ones.

---

3Example: If hypothetically there was any significant time lag between the pressure at the location of the reference pressure transducer and at the paint surface itself, this would be reflected in $H_{\text{sensor}}(\omega)$
4.3 Transfer function for light sensors

4.3.1. Reference illumination and photo detection

If $H_{\text{sensor}}$ is to be determined, $H_{\text{paint}}(\omega)$ must be eliminated from the system. This is done by carrying out the experiment with an reference source of illumination and a reference light sensors. The reference illumination source must be able to deliver a light signal so close as possible to the paint signal in terms of spectral content, intensity, intensity amplitude and offset. Furthermore it must be possible to modulate a sinusoidal intensity signal with an a priori known transfer function $H_{\text{illumination}}(\omega)$, preferably $H_{\text{illumination}}(\omega) = 1$. As reference light source a commercial-grade red LED which emits sufficient light within the band of the optical filter, was chosen. The electrical circuit for the LED is seen on Fig. 4.1 on page 27 A. From the function generator a sinusoidal voltage signal is supplied. The input signal is the voltage measured over $R_2$, which is proportional to the current through the LED and presumably the emitted intensity, which is what has to be shown. The amplitude and offset of the intensity can be scaled by adjusting the aperture between the LED and the light detector. The typical iPSP signal has a ratio between amplitude and offset (AOR) of the intensity signal of 2-4%. Because of the limited dynamic range of the function generator and the cut-in voltage of the LED, the lowest ratio which could be attained, while keeping a well-resolved signal, was approximately 15%. This was realized by minimizing the AOR for the input voltage, and using the value of $R_1$ and $R_2$ to ensure that the LED was operated at proper conditions, and an aperture to control the absolute intensity level at the detector. A reference photo diode (PD) which was known to be very fast-responding, was used as light detector for this test. Its disadvantage was a very low light sensitivity, rendering it unsuitable for PSP measurements. To yield sufficient signal, it had to be used without aperture and directly adjacent to the LED. The transfer function between the LED current (assumed in a linear relationship with the intensity) and the voltage from the PD (C on Fig. 4.1 on page 27) is presented on Fig. 4.10 on the next page. It is seen that $H_{\text{system}}(\omega)$ is virtually unity. This means that

$$g(\omega)_{PD} = g(\omega)_{LED} = 1 \quad \text{and} \quad \Delta \phi(\omega)_{PD} + \Delta \phi(\omega)_{LED} = 0$$

(4.8)

Other solutions to (4.8) than $\Delta \phi(\omega)_{PD} = \Delta \phi(\omega)_{LED} = 0$ would violate the causality principle, and other solutions than $g(\omega)_{PD} = g(\omega)_{LED} = 1$ are extremely improbable for two different, independent instruments. Therefore it can be concluded that the transfer function for the LED circuit is unity. That the LED acted linear was ascertained by considering the output signal, which is virtually perfect sinusoidal, which would not be the case if the system contained a nonlinear component, referring to the argumentation earlier in this section.

4.3.2. Direct modulation of PMT signal results

The experiment is repeated, but with the PMT as light sensor instead. Since the LED is shown to be ideally responding, the resulting transfer function is due to the PMT
only. To illustrate the dynamics of the measurement system, two different values of the terminator ($R_3$ on Fig. 4.1 on page 27) have been tested for PMT 2, up to 1000 Hz. The transfer function is seen on Fig. 4.11 on the next page. Transfer functions for a first-order system (See App. A.7 on page 103) have been fitted to the data, and it is seen that the quality of the fit is good. Since the time constant is proportional with the terminator value, PMT circuit acts as a serial capacitive circuit, which is a first-order system with $\tau = RC$, where $R$ is resistance and $C$ capacitance. This was observed for both PMT’s but PMT 2 had clearly the smallest capacitive effect. This implies that in order to attain a small response time the terminator has to be as small as possible small. On the other hand to get a strong signal to the signal recorder the terminator has to be large. The current of the signal is limited by the rated maximal anode current of the PMT, while the maximal anode voltage is determined by the voltage limits of the signal recorder. Based on these considerations the terminator values were chosen to be 300kΩ for PMT 1 and 100kΩ for PMT 2. For these choices, the transfer function for PMT 2 are seen on Fig. 4.12 on page 42 for low and intermediate frequencies. It is seen that at intermediate frequencies the PMT acts almost ideally. Except for an apparent interference with the grid frequency at 50 Hz, the approximation $g(f) = 1$ and $\Delta \phi(f) = 0$ is valid. For intermediate frequencies PMT2 can thus be used without correction. For low frequencies this is not the case, as the behavior in the sub-Hertz domain the characteristics of PMT 2 is suddenly not ideal any more, but still reproducible enough to do a correction for low-frequency measurements. However it turns out that PMT 1 acts close to ideally at low
4.3 Transfer function for light sensors

Figure 4.11.: Effect of circuit termination on light sensor frequency response for PMT 2. The black lines are analytical solutions for first-order systems. $\tau$ for 300 kΩ termination has been found by LS fitting of the phase signal. For 100 kΩ one third of this value is used.
4.3 Transfer function for light sensors

frequencies, while it at intermediate frequencies exhibits visible gain drop (≈1% at 100 Hz) and phase shift (≈3° at 100 Hz). Whenever possible PMT 1 measurements is therefore used for low frequencies and PMT 2 measurements for intermediate. The results of Fig. 4.12 have been reproduced and fitted with polynomials of fourth order. The measurement points all lie within 0.5% of the fitted value for the $g(f)$ and 0.1° for the phase shift. The deviations from the fitted expressions were within 0.5% for the gain and within 0.2° for the phase shift for both PMT’s. In the following data reduction, correction with the sensor transfer function is for generality always conducted, even in cases where it the effect is vanishing.

![Figure 4.12: Transfer function for the PMT 2. The PMT acts as an ideal instrument at intermediate frequencies, while at low frequencies correction is necessary.](image)

- (a) $g(f)$ vs $f$ Hz
- (b) $\Delta\phi(f)$ vs $f$ Hz
- (c) $g(f)$ vs $f$ Hz (Repeat 1, Repeat 2, Polynomial fit)
- (d) $\Delta\phi(f)$ vs $f$ Hz (Repeat 1, Repeat 2, Polynomial fit)
4.3.3. Characteristic parameters to characterize a transfer function

To keep the overview, a few scalar parameters to characterize the frequency response of the paint are chosen. For analytical systems the frequency response is uniquely determined by these parameters, as is the case for a first-order system whose behavior is defined by its time constant \( \tau \). Since no single model has been found to represent the paint behavior accurately for all cases, general parameters which can be used for any reasonably behaved dynamical system is used. The following parameters can be used for coarse qualitative comparison and benchmarking:

- **Upper frequency,** \( f_{ug} \) or just \( f_u \). Defined as the lowest frequency for which \( g(f) = 0.9 \). This is in accordance with the use of the term in [18]. It is practical furthermore to have this in the form of a characteristic time associated with the upper frequency. We define this time as \( \tau_g = 1/f_u \). The upper frequency is a measure of how fast pressure fluctuation one can measure while having an error of less than 10 percent on the uncorrected pressure amplitude.

- **Upper frequency for the phase shift,** \( f_{u,\Delta\phi} \) Defined analogous to \( f_u \), but for when the phase shift crosses -10 degrees, thus being a measure for how fast variations one can measure, without committing a phase error of more than 10 degrees for the pressure signal.

- **Value of** \( g \) and \( \Delta\phi \) at 10 and 100 Hz, denoted \( g_{10}, g_{100}, \Delta\phi_{10} \) and \( \Delta\phi_{100} \) respectively.

On Fig. 4.13 on the next page the parameters are shown graphically for a measured gain signal. Before the parameters are read off by linear interpolation, the raw data is passed through a 3-point median filter to remove the worst outliers, followed by an 11-point Savitzky-Golay filter of first order [19] to smoothen the signal. The upper frequency \( f_u \) is typically the one parameter used for characterization of frequency response of non-first-order systems. However the phase is typically just as or even more important. Furthermore, since the phase shift is measured directly it is not subject to errors originating from normalization and combination of data sets. Since no assumption has been done with regards to the dynamics of the system, there are no relationship assumed between the gain and the phase shift. Also there is no mathematically well-defined interpretation of the time constant, \( \tau \). These parameters must not be confused with those of a first-order system for which their mutual relationship is fixed (See appendix App. A.7 on page 103).

4.3.4. Discussion

The determination of the transfer function of the measurement setup itself was conducted, and it is complete in the sense that almost no shown assumptions have
Figure 4.13: Definition of $f_u, g_{10}$ and $g_{100}$. The example gain function is measured for a PSP sample.
been made underway, which have not been tested and verified. The exception is the assumption that the PMT’s are not so dependent on the amplitude of the intensity, that they have different characteristics for the paint, than for the LED investigation, for which the intensity amplitude was about 5 times higher. Considering the excellent linearity with regards to intensity, which the PMT’s exhibited in the amplitude linearity test, it seems unlikely that it is the case. The transfer function of the light detectors was determined with an uncertainty which can be neglected for the purpose of PSP measurements, however it is important to conduct measurements with care, and stabilize the PMT before acquiring data, as errors in the order of several percent can be otherwise be committed for the gain at low frequencies. To characterize the transfer function of PSP’s different parameters are suggested. It is recommended to use the phase shift rather than the gain curve for comparisons, as the phase shift is not influenced by normalization errors. It is also clear that the faster a paint is, the more sensitive does the value of the upper frequency become. The parameters $g_{10}$ and $g_{100}$ are more stable in this respect, but less general. Since the measured transfer function for PSP generally have different shape, as will be seen in the next chapter, care in using these parameters has to be exercised.
5. Unsteady PSP Characterization

In this chapter the results for parametric characterization tests for the paint is presented. All results are corrected for the transfer function of the light sensor with which it is measured.

5.1. Reproducibility for the iPSP probe

The transfer function was determined for a probe with the original iPSP formulation. At the same time the test was used as a test of reproducibility by repeated measurements for the an identical probe at intermediate frequencies. On Fig. 5.1 the result of three reproduced measurements is seen. The measurements are reproducible, and the noise level is acceptable. The polynomial fits are of 2nd order, based on all three measurements. The coefficients of the polynomial is seen in table Tab. 5.1 on the following page, along with the standard and maximal absolute deviation of the measured data from the fit. The relatively large maximal error is due

---

**Figure 5.1.** iPSP reproduced measurements on the identical sample with identical light sensor

---

1"Noise" for the transfer function is variations in the outcome from the LS fitting of raw signals. It is not directly measured physical noise, but there is a clear connection between the SNR of the raw signals and the quality of the fit.
5.2 Re-synthesis of Paint

While being able to reproduce results for a single paint sample is the prerequisite for any further investigation, it is also of importance that the paint can be re-synthesized with largely identical properties. Even though one would always one to conduct an unsteady calibration of a paint in connection with the experiment where it is used, a given PSP formulation should exhibit reasonably uniform results from one synthesis to another, and not be too excessively sensitive to the relative content of its components and the synthesis procedure.

A re-synthesis experiment was made half a year after synthesis of the original paint. All samples were made after the same recipe as the original iPSP, with the following modifications.

1 The original iPSP sample

### Table 5.1.

<table>
<thead>
<tr>
<th>Function</th>
<th>Coefficient</th>
<th>Error</th>
<th>Maximum Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(f)$</td>
<td>-2.8180054e-6</td>
<td>-2.0947385e-4</td>
<td>1.0034928e+0</td>
</tr>
<tr>
<td>$\Delta \phi(f)$/°</td>
<td>3.0916105e-4</td>
<td>-1.1771931e-1</td>
<td>2.1964666e-1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value</th>
<th>$f_u$/Hz</th>
<th>$g_{10}$</th>
<th>$g_{100}$</th>
<th>$f_{u,\Delta \phi}$/Hz</th>
<th>$\Delta \phi_{10}$/°</th>
<th>$\Delta \phi_{100}$/°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$iPSP$</td>
<td>158°</td>
<td>1.001</td>
<td>0.954</td>
<td>134°</td>
<td>-0.09</td>
<td>-8.5</td>
</tr>
</tbody>
</table>

**Table 5.2.** Characteristic parameters for the original iPSP sample. The values marked with * are extrapolated from the polynomial fit, and should be considered rough estimates.

To a single outlier at 100 Hz, certainly caused by interference with a power grid harmonic. The standard deviation error is quite low. Characteristic parameters for the original iPSP is seen in Tab. 5.2. The response time of the paint is quite good, and the upper frequency can not be determined, simply because the gain does not drop below 0.9 within the measured range. By extrapolation of the polynomial expression, an estimate can be found, and a characteristic time of approximately 6 ms is found. Shock tube measurements made for the same sample in the small shock tube of DLR Göttingen, showed a response time in the order of 1 ms [20]. Since both values are estimated time scales, whose relation to each other is not exactly defined, they are only expected to be in the same order of magnitude. With the present paint one can with less than 10% amplitude error measure pressure variations of 100 Hz frequency. Since the results are reproducible, the transfer function can be used for a reliable correction, as will be done in the next chapter.
5.2 Re-synthesis of Paint

Table 5.3.: Characteristic parameters for the frequency response of re-synthesized iPSP paint

<table>
<thead>
<tr>
<th>Probe</th>
<th>$f_{u,g}$/ Hz</th>
<th>$g_{10}$</th>
<th>$g_{100}$</th>
<th>$f_{u,\Delta \phi}$/ Hz</th>
<th>$\Delta \phi_{10}$/ deg.</th>
<th>$\Delta \phi_{10}$/ deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>-</td>
<td>1.01</td>
<td>0.96</td>
<td>-</td>
<td>-8.85</td>
<td>-8.76</td>
</tr>
<tr>
<td>2, new FIB</td>
<td>48.7</td>
<td>0.98</td>
<td>0.806</td>
<td>43.8</td>
<td>-3.07</td>
<td>-15.3</td>
</tr>
<tr>
<td>2, old FIB</td>
<td>49.6</td>
<td>0.989</td>
<td>0.824</td>
<td>62.7</td>
<td>-3</td>
<td>-12.7</td>
</tr>
<tr>
<td>3, w. US</td>
<td>34.1</td>
<td>0.989</td>
<td>0.751</td>
<td>30.2</td>
<td>-3.91</td>
<td>-17.2</td>
</tr>
<tr>
<td>3, w/o US</td>
<td>64.9</td>
<td>0.997</td>
<td>0.838</td>
<td>62</td>
<td>-1.6</td>
<td>-13.3</td>
</tr>
</tbody>
</table>

2 One sample with newly synthesized FIB, one sample with FIB saved from the original synthesis

3 Two samples with new FIB, one treated with ultrasound, one without.

The purpose of experiment 2 was to isolate the effect of the FIB synthesis or possible aging. The ultrasound treatment makes the paint more homogenous, which enhances the quality of the PSP images, but its impact on the response time of the paint was unknown. The frequency response of these tests are shown on Fig. 5.2 on the following page. The first thing to note is that the original iPSP is markedly faster than any of the others, and that the attempt to resynthesize it has failed. The second point is that there is no effect at all of the age of the FIB. The two curves from investigation 2, lie so close as one could expect for a reproduction measurement of an identical sample. The ultrasound treatment shows the clear effect of making the paint react slower. The likely explanation for this is that the ultrasound breaks up the porous structures in the paint. While this has a beneficial effect on the paint homogeneity, it also reduces the effective diffusivity of the paint, thus lowering the response time. The sample from synthesis 3 which did not receive ultrasound treatment should be identical with the "New FIB" sample from synthesis 2. While this is indeed the case for the phase shift, the gain is a bit too high, if not much. It is certainly possible that this is a real difference between the two synthesis, as the formulation is potentially very sensible to the amount of filler. On the other hand do the two curves have the same slope, and only difference is their behaviour at low frequencies. It is possible that the difference is caused by the PMT not yet being fully stabilized. The similarity of the corresponding phase signal speaks for this explanation. However, there the difference is small, and the discrepancy between the original sample and the others are much harder to explain. The samples also in their visual appearance (color and homogeneity), as well as the durability of the surface, exhibit marked differences. It is therefore clear that the original iPSP sample is qualitatively different, but alas the reason for this is unknown. In table Tab. 5.3 characteristic parameters for the above experiments are shown. It should be mentioned that even the markedly slower re-synthesized paints have an acceptable frequency response, maintaining about 80% of the signal at 100 Hz.
5.2 Re-synthesis of Paint

5 Unsteady PSP Characterization

Figure 5.2: Frequency response for the series of re-synthesized iPSP.
5.3. Frequency Response dependency of Thickness

As shown in the theory section, the frequency response of a PSP governed by diffusion, has the characteristic diffusion time $\tau = \frac{h^2}{D}$, and is thus sensitive to the layer thickness. It has been found that this relation can be generalized to porous PSPs, for which the characteristic response time under microscopic assumptions with regards to the nature of the porosity, can be described as $\tau \propto h^a$ where $a \leq 2$ [1, 21]. The value of $a$ is highly dependent on the PSP formulation.

In practice a paint layer on a model can hardly ever be applied with perfectly uniform thickness, so this dependence on thickness poses a serious challenge. It is clear that the implication of this on the measurements and the analysis of iPSP would be huge, and the method rendered highly complicated at best and practically impossible at worst, if this relationship should be taken into account.

Two main strategies can be employed to minimize the thickness influence. First and foremost, if $\tau$ is orders of magnitude smaller compared to the relevant timescales of the experiment, the influence of $h$ can be neglected anyway. In addition, for very porous paints the power-law exponent tends to be small [1]. Since the almost all contribution to the luminescent intensity from the paint come from within a couple of optical thicknesses from the surface, a paint where the $\sigma^* << 1$ is expected to be more insensitive to thickness variations. In this investigation a series of eight paint samples with 30% FIB and 70% TiO$_2$ is investigated. The samples have been made in two series of each four samples, and the number of spraying operations has been varied for the samples in order to obtain different layer thicknesses, ranging from the thinnest to the thickest possible.

5.3.1. Thickness measurement

The thickness of the paint layer was measured by means of eddy current (EC) testing. This measurement technique is based on the fact that a time-varying magnetic field generates eddy currents in a conducting solid. This can be utilized for measuring the thickness of an insulating layer on top of an electrical conducting base. An AC coil is placed above the surface and the generation of eddy current is measured, by measuring the change in the coils impedance. With the proper calibration this can be used to deduce the distance from the coil to the conducting surface, and thus the layer thickness [22, 23]. For the present measurements a Quanix 7500 thickness gauge has been used. Each sample had its thickness determined as the average of three repeated measurements at five different spatial positions of the sample. The standard deviations were used as uncertainties, and in table Tab. 5.4 on the following page the thicknesses and measurements are summarized. Especially the two thinnest samples are subject to large uncertainties. This is both due to the resolution of the thickness gauge, which is 0.1$\mu$m, spatial variations in thickness and handling of the gauge. Even though the values are very uncertain, the order of the samples is believed to be correct, as it does
5.3 Frequency Response dependency of Thickness

5.3.1. Table 5.4:

<table>
<thead>
<tr>
<th>h / µm</th>
<th>U(h) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>65</td>
</tr>
<tr>
<td>0.39</td>
<td>57</td>
</tr>
<tr>
<td>0.71</td>
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<td>1.6</td>
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<td>2.4</td>
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<td>3.5</td>
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</tr>
<tr>
<td>3.9</td>
<td>24</td>
</tr>
<tr>
<td>5.7</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 5.4.: Thickness measurement and relative uncertainties

<table>
<thead>
<tr>
<th>Gain function, g</th>
<th>Phase shift function, Δφ</th>
</tr>
</thead>
<tbody>
<tr>
<td>h / µm</td>
<td>f_{u,g} / Hz</td>
</tr>
<tr>
<td>-------</td>
<td>--------------</td>
</tr>
<tr>
<td>0.15</td>
<td>86.2</td>
</tr>
<tr>
<td>0.39</td>
<td>55.4</td>
</tr>
<tr>
<td>0.71</td>
<td>56</td>
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<tr>
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<td>30.3</td>
</tr>
<tr>
<td>2.4</td>
<td>7.78</td>
</tr>
<tr>
<td>3.5</td>
<td>8.39</td>
</tr>
<tr>
<td>3.9</td>
<td>1.66</td>
</tr>
<tr>
<td>5.7</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Table 5.5.: Characteristic parameters for the frequency response for different paint thicknesses

match with the visual impression. For the thinnest samples, the aluminum base can be seen dimly through the paint, whereas the thickest layers are completely opaque.

5.3.2. Results

On Fig. 5.3 on the next page the gain and phase shift is seen for the 8 samples. The frequency response has been measured at low and intermediate frequencies, and the data merged as described in section. Firstly it is seen that the qualitative order of the frequency responses is as expected. The thinner the paint, the faster the response, which is valid for both gain and phase shift. The phase shift is virtually zero for all thicknesses at \( f = 0.1 \text{ Hz} \), why the quasi-static assumption seems reasonable.

For the four thickest samples, the phase shift seems to converge to the same curve. For \( f > 40 \text{ Hz} \), these four phase shift curves are close to identical, whereas they differ more at lower frequencies. For the gain, the difference between the same five curves are more distinct, though their shape is similar. The thinnest samples show visible differences both for gain and phase shift. In Tab. 5.5 the characteristic parameters for transfer functions are summarized. The characteristic time, both as \( 1/f_{u,g} \) and \( 1/f_{u,Δφ} \) is
5.3 Frequency Response dependency of Thickness

Figure 5.3.: Transfer function for different paint thicknesses. The used paint is 30%/70% FIB/TiO$_2$
5.3 Frequency Response dependency of Thickness  

plotted against $h$ in the double-logarithmic plot of Fig. 5.4.

It is seen that no good power-law fit is available, however if a power-law has to be applied a power of two is quite reasonable, if the two thinnest samples for which the thickness is very uncertain are excluded. This suggests that the paint can be described qualitatively by the diffusion model. But some aspects of the paint behavior is not explained well by it. Firstly, according to the diffusion model the phase shift curves should be similar in shape and eventually converge to $-45^\circ$, which does not seem to be the case here. Also, it can not explain the insensitivity to thickness at high thicknesses seen from Fig. 5.3 on the preceding page. An attempt to explain these properties as due to the light attenuation in the sample, by applying the diffusion model with optical thickness (DOT) as described in the theory section Sec. 2.3.2 on page 13, can be made. The model was implemented numerically for a number of dimensionless optical thicknesses. The algorithm is based on the theory from section Sec. 2.1 on page 3, and sketched in table Tab. 5.6 on the next page. The linear calibration model from (4.4) and $P_1^* = 0.03$ was used. On Fig. 5.5 on page 55 the modeling result is seen together with measurement data for the five thickest samples, in form of a polar transfer function plot and phase shift plot. A important property of the polar plot is that it addresses the shape of the transfer function independently of the time scale.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig54.png}
\caption{Response time for as function of paint thickness. Both a $1/f_{u,g}$ and $1/f_{u,\Delta \phi}$}
\end{figure}
Table 5.6.: Algorithm for determining \( g(\omega^*) \) and \( \Delta \phi(\omega^*) \) numerically with the diffusion model. \( \forall \) denotes a loop. If the modeling does not include modeling of optical thickness, level A is let out and \( \eta^*(x^*) \) is set to 1 at level E

Readers unfamiliar with the concept of a polar plot of a transfer function are referred to appendix App. A.8 on page 104. It is seen that the measured transfer function are not similar, and have a considerably different relation between gaina dn phase shift that the diffusion model predicts. From the model results for different optical thicknesses it is seen, that this qualitative behavior, is predicted by the DOT model. The effect of modeling optical thickness is mainly that the phase shift never converges to \(-45^\circ\) as in the pure diffusion model. Instead it ultimately converges to zero\(^2\), but only after going through a large frequency range of virtually constant phase shift. The smaller the optical thickness, the smaller the magnitude of the phase shift at this level. This can account for the phase shift of the measurements seemingly converging to values higher than \(-45^\circ\). For the measured data in the polar plot (top), the shape of the transfer function can be directly compared to the shape of the family of DOT model transfer functions. For the measured transfer functions, \( h \) is known, and for the modeled transfer functions \( \sigma^* = \sigma/h \) is known. The single value of \( \sigma \) which yields an acceptable correspondence between the \( \sigma^* \) values of measured and modeled curves, is used as an estimate for the optical thickness of the paint. This was assessed to be \( \sigma = 1 \, \text{\mu m} \). This concerns the similarity of the transfer function shape. To obtain the right scaling of the frequency, the same method is used for the phase plot to scale the measured values to the theoretical. The fitting parameter is the time constant, \( \tau \).

Assuming that the the characteristic time scale of the diffusioon model is valid, \( \tau = \frac{h^2}{D} \), a common value of \( D \) is found, which provides the best fit of the measured data to

\(^2\)That \( \omega^* \to \infty \) actually \( \Delta \phi(\omega^*) \to 0 \) can be explained the following way: As frequencies become very high, the concentration profile cannot follow the rapid oscillations except at the very surface. For unattenuated light the intensity from within the layer (which is out of phase) has influence as well, but for optical thickness, the relative weight of the outer layer intensity contribution increases with the frequency.
Figure 5.5: Polar and phase plot for DOT model and the 5 thickest samples. The full curves are DOT model results, and the points are measurements, with $\frac{h^2}{2.5 \times 10^{4} \text{m}^2/\text{s}}$. 

- $\text{DOT, } \sigma^* = 0.1$
- $\text{DOT, } \sigma^* = 0.2$
- $\text{DOT, } \sigma^* = 0.3$
- $\text{DOT, } \sigma^* = 0.5$
- $\text{DOT, } \sigma^* = 10$
- $1 \mu m/h = 0.18$
- $1 \mu m/h = 0.25$
- $1 \mu m/h = 0.29$
- $1 \mu m/h = 0.41$
- $1 \mu m/h = 0.64$
their corresponding theoretical curve, here estimated as \( D = 2.5 \cdot 10^{10} \text{m}^2/\text{s} \). This is a surprisingly low value, as the diffusivity of typical non-porous silicon polymers is in the order of \( 10^{-9} \text{m}^2/\text{s} \) \([1]\), and a porous polymer should be expected to have considerably higher diffusivity. Expected or not, as seen on the phase graph the model does a reasonable job describing the measured data, and predicting the main effects of the sample thickness, even though it does not equally well over the whole frequency range. However, the model is only able to explain the tendencies and qualitative behavior of the curves. The main conclusions are the following:

- High-quality power-law behavior of the response time could not be shown. The trend was a power-law behavior with \( \tau \propto h^2 \), but deviations from this were significant.

- The results can be described qualitatively with the DOT model for \( \sigma = 1 \mu\text{m} \) and \( D = 2.5 \cdot 10^{10} \text{m}^2/\text{s} \). These parameters are fitted, and the physical significance unverified. Though the DOT model provides a qualitative improvement compared to the diffusion it still does predict different curve shapes than measured. It is assessed that the model reflects a physical mechanism, but that the actual mechanisms of the porous paint are much more complicated, than the diffusion model can account for.

- The paint shows invariance for the phase with respect to thickness for \( h \geq 2.4 \mu\text{m} \). The gain is still changing a somewhat. A practical conclusion is that paints should be made as thick as possible, if the response time for the thick paint layer is sufficient for the application.

### 5.4. Frequency Response Dependency on Filler Content

As seen in chapter two, the bottleneck with regards to the response time of PSP, is the diffusion speed in the PSP layer. The transport capabilities of the paint layer is dramatically increased by making it porous, which can be realized in several ways. A well-known way to obtain excellent porosity is to use anodized aluminium as the matrix for the luminophore \([11]\), which gives unsurpassed response times in the order of \( 100 \mu\text{s} \) \([1]\), however this method does not possess the crucial advantages of a sprayable paint, and is practically inapplicable for most models. A wide variety of porous polymers exist, notably poly(TMSP) which has a characteristic response time of less than 10 ms \([24]\). The general problem is that these fast polymers tend to be unstable, and significantly change their properties as function the light exposure, temperature and time. For the iPSP another strategy was chosen, namely to add microparticles to an otherwise relatively slow polymer, in this case FIB, thus providing porosity while in outline retaining the good properties of the polymer.
In the present case an investigation of the amount of filler in the paint, was carried out. The filler used was rutile titanium dioxide, TiO$_2$ microparticles. A series of 11 samples was prepared, consisting of PtTFPP in FIB and TiO$_2$. The mass percentage of TiO$_2$ in the matrix was 0, 10, 20, 30, 40, 50, 60, 70, 73, 77, and 80 respectively. For fast response time, naturally high porosity and thus a high mass percentage of TiO$_2$ is desirable. However for practical application the percentage of polymer must be large enough to provide sufficient adhesion and mechanical stability of the paint layer to withstand handling and aerodynamic forces during measurements. With an insufficient amount of binder the paint layer on a model will simply be torn off by the shear forces in the flow, or be damaged by impacting particles.

### 5.4.1. Results

The results of the measurements are seen on Fig. 5.6 on the following page. It is clearly seen that the fastness of the response is increased for increasing porosity. This correlation can also clearly be seen from Tab. 5.7. The upper frequencies are increasing with TiO$_2$ content, while $g_{10}$ and $g_{100}$ are going towards one and $\Delta \phi_{10}$ and $\Delta \phi_{100}$ towards zero. The paint without any TiO$_2$ is seen to be very slow, as expected for a non-porous paint. Its upper frequency of 0.328 Hz corresponds to a response time in the order of seconds. This might even be considerably slower, as the phase shift at 0.1 Hz is not negligible for the 3 slowest paints. Only the 80% TiO$_2$ paint shows a response which is in the desired order of magnitude.

### 5.4.2. Analysis

On Fig. 5.7 on page 59(top) a polar plot of the transfer functions have been made. It is

<table>
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<tr>
<th>TiO$_2$-percent</th>
<th>$f_{u,g}$/ Hz</th>
<th>$g_{10}$</th>
<th>$g_{100}$</th>
<th>$f_{u,\Delta\phi}$/ Hz</th>
<th>$\Delta \phi_{10}$/ deg.</th>
<th>$\Delta \phi_{100}$/ deg.</th>
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<td>0.368</td>
<td>0.17</td>
<td>0.22</td>
<td>-25.5</td>
<td>-34.4</td>
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<td>0.229</td>
<td>0.97</td>
<td>-28.6</td>
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<td>20</td>
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<td>0.27</td>
<td>0.523</td>
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<td>-36.3</td>
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<tr>
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<td>0.598</td>
<td>0.29</td>
<td>0.659</td>
<td>-20</td>
<td>-33.1</td>
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<tr>
<td>40</td>
<td>0.538</td>
<td>0.62</td>
<td>0.318</td>
<td>1.26</td>
<td>-17.6</td>
<td>-33.2</td>
</tr>
<tr>
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<td>0.375</td>
<td>1.21</td>
<td>-16.3</td>
<td>-30.6</td>
</tr>
<tr>
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<td>1.01</td>
<td>0.715</td>
<td>0.419</td>
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<td>-13.4</td>
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<tr>
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<td>0.99</td>
<td>0.902</td>
<td>-</td>
<td>-2.1</td>
<td>-7.51</td>
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</tbody>
</table>

Table 5.7.: Characteristic parameters for the frequency response for TiO$_2$ content
5.4 Frequency Response Dependency on Filler Content

Figure 5.6: Frequency response for different TiO$_2$ concentrations

- $g(f)/dB$ vs. $\log(\frac{f}{Hz})$
- $\Delta\phi/\text{deg.}$ vs. $\log(\frac{f}{Hz})$
Figure 5.7.: Polar plot for the transfer function for varying TiO$_2$ content.
seen that all transfer functions are overlaid, which suggests that they are similar with respect to a characteristic time scale, and that the optical thickness mechanism plays no big role. This is supported by the fact that the thicknesses of these paints are all in the range from $3 - 13 \mu m$, thus thicker than the thicknesses from the previous section where thickness had a significant effect. In this case it is of course complicated by the fact that the composition of the samples are different, and thus also the optical thicknesses. In fact, none of the theoretical models describe the curve shape very well, but from the Bode plot of the gain, Fig. 5.7 on the previous page (bottom), it is seen that the similarity with respect to a time scale is a fact. The frequency has been scaled individually with a value for $\tau$ for each graph, with $\tau$ normalized to one for the pure polymer sample, but since $\tau$ has no direct interpretation for any of the present models, its absolute magnitude is arbitrary. For a perfect signal, using any characteristic time scale, e.g. $1/f_{u,\Delta \phi}$, should principally give the non-dimensional form. This was tried, and the result was close but not equal to that of Fig. 5.7 on the preceding page (bottom). Instead manual fitting was employed to yield visibly better results. The similarity is striking, except for the 0, 10 and 20 % TiO$_2$, which have excessive gain at low frequencies. This is most likely due to the fact that these, the slowest, paints have not yet reached a phase shift of zero, even at the lowest frequency. Therefore the normalization of the gain is not done at $\Delta \phi = 0$, which causes an unphysically high gain level at low frequencies. It can also not be ruled out that the thickness plays a role, as the samples for 10 and 20 % TiO$_2$ are thin compared to the other samples (See appendix App. D on page 112 for sample thicknesses). The sensitivity to thickness is assumed to be small since the samples are thick compared to the thickness experiment. This is an assumption, since because of the composition of the samples, the optical thickness will not be constant for this measurement series. But as already seen, these effects do not seem to have much influence. On Fig. 5.8 on the following page the fitted characteristic timescale, $\tau$ (normalized with its value for the sample with no TiO$_2$), is shown as function of the TiO$_2$ content, along with the sample thickness normalized with respect to the thickest sample in the series (30% TiO$_2$, 13 $\mu$m). The response time is seen to drop for increasing TiO$_2$ percentage. The relation is remarkably close to linear for the measured points if the point for 0% TiO$_2$ is excluded, but it is clear that the linear form of the relation must break down for $\tau$ approaching 0. It is seen that the trend does not seem to correlate significantly with the thickness of the samples. An identical experiment was carried out for another fluoropolymer called IBM$^3$, for intermediate frequencies. The results were very similar, and in Fig. 5.9 on page 62 the phase shift at 10 and 100 Hz for the FIB and IBM polymer are shown. From the figure it can be seen that both the trend and the value of the phase shift are close to identical. It is therefore made plausible that the effect of adding filler is relatively independent of the polymer.

---

3Short of poly(isobutylmethacrylate-co-trifluoroethylmethacrylate) [25]
Figure 5.8: Characteristic response time for PtTFPP/FIB and different TiO$_2$ concentrations. $\tau/\tau_0$ is found from the scaling of Fig. 5.7.
Figure 5.9.: Phase shift at 10 and 100 Hz for FIB and IBM with TiO$_2$
5.5. Frequency Response dependency on reference component content

The role of the reference component NP-108-62 was investigated with a series of four IBM polymer samples with 0, 20, 40 and 60 % reference component respectively. On Fig. 5.10 on the following page the result is seen. As seen does the reference component cause virtually no difference between the frequency responses, which are all very slow. Remark that the phase shift already in the order of $-20^\circ$ at 1 Hz, why a low-frequency investigation, would result in lower gain-values than shown here. This was done for the pure FIB sample on Fig. 5.6 on page 58, and the difference is seen to be very modest. From the phase shifts of Fig. 5.10 on the following page it is seen that the frequency response for the four curves is very similar, and it is concluded that the addition of NP-108-62 has no significant effect on the time response of the paint, except lowering the response time insignificantly.
Figure 5.10.: Frequency response for reference compound concentrations for IBM polymer
5.6. Discussion and Summary of paint results

The main conclusion on the paint results was that the iPSP sample showed a suitable frequency response for measuring pressure changes in the order of 100 Hz. The frequency response was reproducible with sufficient accuracy to perform a correction of measured pressure data. It also became apparent that the paint samples had markedly different characteristics. It can not be explained, but only be used to point out that the synthesis of PSP is in no way fully understood with respect to the implications of recipe and synthesis procedure on the final paint formulation, especially regarding its microscopic structure. Since the original sample was measured to be 20 µm thick, and its reproductions ranging from 7-14 µm, thickness can not explain that the original sample is faster, if anything it should be the other way around. It was found that thickness have a considerable influence on the response time, but that this influence can be minimized by making the layer as thick as possible, which alas also maximizes the response time. A good unsteady PSP should thus either be so fast that no correction is required in any case and the thickness effect thus irrelevant, or it should be be applied with sufficient thickness such that the transfer function can be assumed to be converged to the value for high thickness. Comparing the thickest probes of 5.5 (top) with the all the measurements of 5.7 (top), it is seen that the curves are very similar, this also supports the suggestion that beyond a certain thickness, the transfer functions become similar. These mechanisms can be qualitatively understood by the notion of a limited penetration depth into the paint layer. The DOT model is in outline able to account for the behavior of the samples in the thickness investigation, even though the large uncertainties on the thickness data make it more of a plausibility study, than a waterproof demonstration. It is clear that the effect of optical thickness is undoubtedly true in for the upper limiting case, namely an infinitely thick layer. For the notion to have any significance here, however, it is required that the paint layer can be made thicker than its optical thickness. In the present case the modeling of optical thickness provides a marked better description of the transfer function shape than the the pure diffusion model, as was shown on Fig. 5.5 on page 55. However there are still some major discrepancies between the model and the observations, and it the model thus fails to make a real quantitative description of the influence of thickness. Among possible explanations for this is that the layer thickness is not from uniform. For a layer where the transfer function, \( H = H(\omega, h) \), is a function of the layer thickness \( h = h(x', y') \) where \( x' \) and \( y' \) are spatial coordinates, then because of the linearity of the Fourier transform, \( Y(\omega) = \int H(\omega, h(x', y'))dx'dy'X(\omega) \). The transfer function which is measured in case of varying thickness is therefore an integral over a local transfer function which depends on the local thickness. This extends the accessible transfer functions of the DOT model to linear combinations of those for uniform layer thicknesses. It is also very likely that the transport mechanism of the porous layer is simply too qualitatively different from diffusion to be explained by the notion of an effective diffusivity. It is a complicated question, and too much is unknown about the microscopic structure of the
5.6 Discussion and Summary of paint results

paint to give a clear answer to this question here.

For the investigation of TiO$_2$-content, all samples are thicker than 3 $\mu$m and all show similar curve shapes, which suggest that the thickness differences are without significance. It was shown that the more filler, the faster the paint responds. For practically applicable paints, however, no more than around 70% filler can be used, as it is the polymer which keeps the layer together. For tests not shown here filler percentages of 90% gave transfer functions with almost no gain drop and phase shift up to 120 Hz, but the paint was simply so brittle that it could easily be removed with a nail. These paints are therefore of no practical interest. The paint for 70% TiO$_2$ is essentially iPSP without the reference component, however the iPSP with reference component is significantly faster. This is rather surprising, since the reference component in itself makes no such difference according to Sec. 5.5 on page 63. What seems to be the case is that in combination with TiO$_2$, the NP-108-62 adds positively to the porosity, in a way it does not do independently, or in the IBM polymer. The iPSP contains 16% FIB, and its response is similar to what would be expected if the remaining 84% were TiO$_2$, and it could seem that it acts as a filler in this case, but without making the paint extremely brittle. It is remarkable that the iPSP sample which is the thickest of them all, is also the fastest responding, while consisting almost 50% NP-108-62, which alone had only a small positive effect on the response time. The transfer function were determined up to $f = 120$ Hz. For higher frequencies the amplitude limitation of the pressure chamber meant that the SNR of the PMT signal rapidly became too low for the sine-fitting algorithm to yield meaningful results.
6. Application for a Wind Tunnel Experiment

To evaluate the practical use of iPSP for industrial measurements a wind tunnel experiment was carried out for an airfoil coated with the iPSP paint. The measurement campaign had multiple purposes, but in the present thesis a subset of the measurements will be used to present and evaluate the iPSP analysis procedure and to compare the results with conventional unsteady pressure measurements.

6.1. Purpose and Description of Experiment

The measurement campaign were conducted as a cooperation between the Department of Aeroelastics and Department of Experimental Methods of DLR Göttingen, and served both as a transonic flutter investigation for the first group, and as a test of the unsteady PSP measurement system for the latter. It is from the latter point of view this experiment will be seen in the following. The purpose was not only to test the paint itself, but the whole measurement system developed for unsteady PSP including illumination, cameras, control, software, measurement procedures etc.

6.2. Hardware

6.2.1. The Transonic Wind Tunnel in Göttingen

The experiment was carried out in the transonic wind tunnel (DNW-TWG) of German-Dutch Wind Tunnels (DNW) in Göttingen. The DNW-TWG is a pressurized (0.3-1.5 bar) continuously-driven facility with interchangeable test section. Depending on the test section the Mach number can range from 0.3 to 1.2. The present experiments was carried out with a 1 m×1 m test section with adaptive walls. The roof and floor in this section are flexible and can by means of 2×23 electrical actuators be shaped to minimize vertical wall interference. A sketch of the DNW-TWG is seen on Fig. 6.1 on the next page.
6.2. The NLR7301 Profile

The airfoil used for the test is the supercritical\(^1\) airfoil NLR7301 [27, 28]. The profile is made of carbon fiber material coated with a white screen layer, has a span of 1.00 m and a chord length of 0.30 m (Fig. 6.2 on the following page). At the midsection the profile is equipped with two staggered rows of pressure taps equipped with micromechanical pressure transducers (Kulite). The staggered arrangement makes it possible to fit 35 sensors at \(x/c\) positions between 0.05 and 0.85, at the expense of having to make the assumption that the pressure distribution along the two rows are identical. The left 60 percent of the profile is coated with iPSP. The remainder of the profile is covered with TSP for transition detection, an investigation which will not be covered in the present work. Instead the flow temperature is measured by a PT100 temperature sensor\(^2\) embedded mid-chord on the suction side, close to the edge of the profile. The NLR7301 is mounted such that its pitching motion is controlled by a hydraulic actuator.

---

\(^1\) A supercritical airfoil design seeks to increase the drag divergence Mach number, \(M_{ad}\). This is the freestream Mach number for which the drag starts to increase dramatically due to effects associated with a transonic shock (Chiefly shock-induced separation and transition). Supercritical airfoils have a blunt leading edge, high thickness and a highly cusped trailing edge. The shock will typically be weaker and closer to the trailing edge than for conventional foils. The beneficial effects are only obtained in a narrow envelope for Mach number and angle of attack. An good introduction to basic supercritical airfoil theory and design is given in [26].

\(^2\) PT100 is a temperature sensor, which uses the resistance of a Platinum wire or, as in this case, foil, to gauge the temperature. The characteristics of a Pt100 sensor is regulated by DIN IEC 751.
6.2.3. Lamps and Cameras

The excitation light is provided by a UV lamp, EP1. The EP1 has three high-pressure Xenon bulbs with, each connected to its own optical fiber, whose other end is mounted into the TWG wall for profile illumination (Fig. 6.3 on the next page). The lamp is hermetically sealed, and maintains atmospheric pressure internally to ensure sufficient cooling and to protect the bulbs. For the present measurements the EP1 delivered light pulses of 100 $\mu$s duration, with an effect of up to 10 kW white light. The maximum repetition rate for these pulses is 0.2 Hz. Optical bandpass filters are inserted in the lamp, such that the emitted wavelength is around 375 nm. The pressure and reference images are acquired by two cooled 14-bit CCD cameras (PCO-4000), each with a optical bandpass filter mounted in front of the objective (see Tab. 6.1).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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<td>630-800</td>
<td>nm</td>
</tr>
<tr>
<td>Reference filter transmission</td>
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<td>475-590</td>
<td>nm</td>
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<td>Images per measurement point</td>
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Table 6.1.: Experimental parameters for NLR7301 Experiment
The cameras are triggered by the same signal, and the exposure time is as much as 1 second, such that the effective exposure time is fully determined by the flash duration and the luminescent lifetime of the paint.

![DLR-EP1](image1.png) ![Camera arrangement](image2.png)

**Figure 6.3.**: a) The DLR-EP1 lamp. The light guides are seen at the left side. b) The reference camera (left) and the pressure camera (right) mounted on the TWG. Also seen are the three excitation light guides.

### 6.2.4. Measurement

The wind tunnel walls were adapted to minimize the wall interference at the mean angle of attack, $\alpha = \overline{\alpha}$. Before starting the wind tunnel wind-off images for both pressure and are acquired. The wind tunnel is started, and when the flow is steady, the measurement is initiated. The profile is subject to forced oscillations

$$\alpha(\Psi(t)) = \overline{\alpha} + \alpha_{amp} \sin(\Psi(t)) \quad (6.1)$$

where the phase of the motion, $\Psi(t)$, is

$$\Psi(t) = \text{rem}(2\pi f_{osc} t, 2\pi) \quad (6.2)$$

where rem is the remainder by division, such that that $0 \leq \Psi < 2\pi$. For the experiment covered in this report, the oscillation frequency, $f_{osc}$, was 30 Hz. The parameters to characterize the flow are summarized in Tab. 6.2 on the next page.

Every time $\Psi(t) = 0$ a signal is sent from the oscillation actuator. To measure at a specific phase, $\Psi_0$, a delay of $\Delta t = \frac{\Psi_0}{2\pi f_{osc}}$ added to the trigger signal. For a phase change of 10 degrees $\Delta t$ is thus 926 $\mu$s, and due the short time and required precision, the delay is realized by means of a dedicated hardware solution. The EP1 when fully charged and ready, is triggered and returns a signal when the flash is activated, which is then used to synchronize the image data with the profile, flow and Kulite data. This is sketched on Fig. 6.4 on the following page This is repeated four times for each phase to verify the periodicity of the pressure field, and to reduce noise by ensemble averaging.
Table 6.2.: Flow Parameters for NLR7301 experiment

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</tbody>
</table>

Figure 6.4.: Example of angle of attack and the synchronization pulse as function of time. Based on the time for onset of the synchronization pulse and the blitz duration of the EPI is used to determine the time at the middle of the blitz, and corresponding $\alpha$ and $c_p$ values are determined.
6.3. Analysis Procedure

In this section the analysis procedure for the PSP will be explained. The analysis is facilitated by the in-house developed DLR software ToPas (Three-dimensional Optical Pressure Analysis System) [29–31], which is specifically designed for PSP measurements and contains a lot of advanced functionality. The amount of data to be processed is huge, and here the procedure will be illustrated for one individual measurement point\(^3\). With each measurement point the following PSP images are associated, for reference and pressure camera respectively:

- \(I_{\text{r,dark}}\) \(I_{\text{dark}}\), Dark images, acquired without excitation light.
- \(I_{\text{r,wo}}, I_{\text{wo}}\), Wind-off images, acquired with excitation light but without wind.
- \(I_{\text{r}}, I\), The wind-on images, with excitation light and with wind.

In the above notation \(\dagger\) denotes that the image is raw, i.e. no operations has been done on the pixel values. A matrix notation (bold font) is adopted for images, element \((i, j)\) in an image matrix being the intensity value of pixel \((i, j)\). In principle this means 6 images per measurement point, however due to limited tunnel time the wind-off and dark images are not reproduced at each measurement point. Instead one set of each is acquired at the beginning of the measurement series, and used for all measurements in this series. For the dark image this can be justified as long as it is actually dark, in the sense that virtually no information about the model can be extracted from it and the only features of the image is due to noise and CCD inhomogeneity. This, however, should always be the case for the dark image. For the wind-off image the practice can be adopted as long as the model position does not change too much within the measurement series, since the analysis is based upon the assumption that the relative excitation light distribution is identical on wind-on and wind-off image. In the present case where the profile does only pitch up to 0.6 degrees from its mean pitch value, this is probably a good assumption. On Fig. 6.5 on the next page an example of \(I_{\text{dark}}\) and \(I\) for one measurement point is seen. The dark image is seen to reflect some CCD inhomogeneity, but the intensity level is very low. On the pressure image, it is seen that already for the uncorrected image, the features of the pressure distribution can be sensed, which is a nice feature since this means that qualitative pressure distributions are available online.

\(^3\)By measurement point is understood data acquired at one specific time, i.e. 2 PSP images (+2 wind off images), and corresponding wind tunnel, and Kulite data. By measurement series is meant a number of consecutive measurement points which is part of the same investigation
6.3 Analysis Procedure

6.3.1. Dark image subtraction

As a first step the dark image is subtracted from the wind-on and wind-off images

\[ I_{r,\text{wo}} = I_{r,\text{wo}} - I_{r,\text{dark}} \]
\[ I_{b,\text{wo}} = I_{b,\text{wo}} - I_{b,\text{dark}} \]
\[ I_{r} = I_{r} - I_{r,\text{dark}} \]
\[ I_{b} = I_{b} - I_{b,\text{dark}} \]

By performing this step the level of the camera dark noise is removed. Furthermore a 0th order correction is made for intensity differences caused by inhomogeneities in CCD chip. At this stage also image averaging of the four images acquired is done. On Fig. 6.6 on the following page an example of \( I_{r,\text{wo}}, I_{b,\text{wo}}, I_{r}, \) and \( I_{b} \) is shown. At the wind-off condition (a) and (c) the relative intensity distribution on the pressure and reference camera is similar as it should be, except for some edge effect for the reference camera. The reference images reflects the distribution of excitation intensity, while the pressure images reflect the combined effect of excitation intensity and pressure distribution.

6.3.2. Filtering

The 2D images are eventually filtered. The type of filter applied will always depend on the concrete experiment, but typically a median filter will be used. Let \( \xi_{M \times N} \) be a M×N median filter, and let it operate on the matrix element \( I_{i,j} \). Then

\[ \xi_{M \times N} (I_{i,j}) = \text{median} \left( \begin{bmatrix} I_{i-a,j-b} & \cdots & I_{i+a,j-b} \\ \vdots & \ddots & \vdots \\ I_{i-a,j+b} & \cdots & I_{i+a,j+b} \end{bmatrix} \right) \]
6.3 Analysis Procedure 6 Application for a Wind Tunnel Experiment

Figure 6.6.: The intensity images, averaged and with dark image correction. Colormap goes from 50 counts (blue) over green, yellow and red to 12700 counts (white)
where

\[ a = \frac{M - 1}{2} \quad \text{and} \quad b = \frac{N - 1}{2} \]

In the present case a $7 \times 7$ median filter has been used. A median filter has the useful property of being insensitive to outliers and edge regions. As an example of the opposite an averaging filter or Sawitsky-Golay smoothing filter [19] would smear out the values in the edge regions, rendering values near the edges unusable.

### 6.3.3. 3D Alignment

In order to do the intensity correction, the images on Fig. 6.6 on the previous page must be aligned, by mapping the 2D image data onto the 3D grid of the model. The 3D grid is typically available a priori in form of a CAD file. Before the being built into the tunnel model is equipped with a set of markers. The markers are small dots put onto on the surface of the model with a marker pen, and their 3D positions subsequently carefully measured with a 3D digitizing system. An example of such markers and digitizing system is seen on Fig. 6.7. For the current measurements a significantly more advanced setup than the one shown on Fig. 6.7 was used, but the principle is identical. The measured marker positions are expressed in the coordinate system of the 3D model grid. The markers have to be small in order to for their positions to be well-defined, while still being large enough to be clearly recognizable on the image. In general the higher number of markers, the more precise the 3D alignment will be, the disadvantage of using more markers is chiefly the increased amount of work during 3D digitizing. The markers are distributed such as to represent the geometry of the model as well as possible. In the present case, where the model is simple and the markers can be distributed well and be visible all the time, 9 markers

![Figure 6.7:](image_url)
are sufficient. The marker position already being known in 3D, are now determined on
the 2D images. The identification of the 2D markers is in principle simple. To
accurately determine the position of a single marker, a rectangular search pixel region
is defined around the marker, and a correlation mask used. A correlation mask is in
this context a continuous function $I_{\text{mask}}(x, y)$, which is supposed to simulate a typical
marker intensity pattern. In TOPAS several types parametrized masks are available,
and can be chosen to suit the concrete application. Typical masks can be of the form

$$I_{\text{mask}}(x, y) = 1 - \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \quad (6.5)$$

or

$$I_{\text{mask}}(x, y) = \begin{cases} 0 & \text{for } \sqrt{x^2 + y^2} \leq r_1 \\ \frac{\sqrt{x^2 + y^2}}{r_2 - r_1} & \text{for } r_1 < \sqrt{x^2 + y^2} < r_2 \\ 1 & \text{for } \sqrt{x^2 + y^2} \geq r_2 \end{cases} \quad (6.6)$$

Within the search region the two-dimensional cross-correlation, $(I_{\text{mask}} * I_{\text{image}})(x', y')$, bet
between the image pixel values and a marker mask is found:

$$(I_{\text{mask}} * I_{\text{image}})(x', y') = \sum_i \sum_j I_{\text{mask}}(x_i + x', y_j + y')I_{\text{image}}(x_i, y_j) \quad (6.7)$$

The $(x', y')$ for which $(I_{\text{mask}} * I_{\text{image}})(x', y')$ attains its maximal value is the value where
the marker fits the mask best. If the marker has sufficient size (at least a couple of
pixels in diameter), the method has sub-pixel accuracy, and share many similarities
with correlation methods used for Particle Image Velocimetry [3]. The whole
marker-finding procedure is semi-automatized as discussed in appendix App. C on
page 110. From the 3D digitizing the marker positions are accurately known in the
coordinate system of the 3D model. Now the intensity values of the 2D images has to
be mapped onto the 3D grid of the model. This procedure can be formulated as a
general minimization problem (see appendix App. B on page 106). Let $(\tilde{x}_i, \tilde{y}_i)$ denote
the coordinates of the $i$th marker on a 2D image, and $(x_i, y_i, z_i)$ the measured 3D
dimensions. The alignment procedure can be described as

$$\begin{align*}
\begin{cases}
\tilde{x}_i \\
\tilde{y}_i 
\end{cases} &= \Psi(c) \begin{cases}
x \\
y \\
z
\end{cases} 
\end{align*} \quad (6.8)
$$

where $\Psi(c)$ is the mapping operator, which depends on a number of DOFs represented
by the vector $c$. The best alignment is the one that minimizes the expression

$$\sum_{i=1}^N \left\| \Psi(c) \begin{cases} x_i \\
y_i \\
z_i \end{cases} - \begin{cases} \tilde{x}_i \\
\tilde{y}_i \end{cases} \right\| \quad (6.9)$$
Hence, it is a minimization problem of the form of (B.1). Implemented in ToPas is a 11-DOF optical model. It contains the degrees of freedom

\[
c = \begin{cases}
\alpha & \text{Rotation around x} \\
\beta & \text{Rotation around y} \\
\phi & \text{Rotation around z} \\
x' & \text{Translation, x-direction} \\
y' & \text{Translation, y-direction} \\
z' & \text{Translation, z-direction} \\
\alpha_c & \text{Perspective correction} \\
d_c & \text{Perspective correction} \\
s_x & \text{Scaling in x-direction} \\
s_y & \text{Scaling in y-direction} \\
a & \text{Parameter for quadratic lens correction}
\end{cases}
\] (6.10)

By the iterative procedure described in appendix App. B on page 106 and App. B.1 on page 108, the set of parameters, \(c\), which yields the least squared error is found. On Fig. 6.8 on the following page the procedure is shown graphically. The 2D images are shown and overlaid is the 3D geometry, based on the current value of \(c\). On Fig. 6.8 on the next page a), \(c\) has an arbitrary initial value, so the alignment is not good, whereas b) shows the aligned geometry. With \(c\) known, every point on the 3D grid can now be associated with a 2D coordinate on the image by means of (6.8). This does not give integer (pixel) values, so the 2D values have to be rounded. As the 2D image resolution is typically much higher than the resolution of on the 3D grid, this does not cause any significant errors. In fact it really at this point the filtering (Sec. 6.3.2 on page 73) of the images is done, so that it needs only to be done for the pixel values, which are actually used.

### 6.3.4. Intensity Correction, Calibration and Offset Correction

The intensity correction can now be applied, as described in Sec. 2.1.2 on page 8, for each evaluation point on the 3D-grid.

\[
I^* = \frac{I \cdot I_{r,wo}}{I_r \cdot I_{wo}}
\] (6.11)

The steady calibration for this test was performed in situ, i.e. with the profile itself in the wind tunnel at no-wind conditions at the calibration pressures and temperatures. The pressures was measured at the wall of the test section, and the temperature by the PT100 sensor. If the wind tunnel is pressurized, this should always be preferred to a pressure chamber calibration, in order to ensure as realistic conditions as possible. In this way any errors introduced by differences in paint, light conditions, equipment and measurement method of pressure and temperature are minimized. A representative area of the model is chosen to provide the basis for the averaging to get one global
Figure 6.8.: Illustration of the alignment procedure. a) shows a bad alignment for typical set of initial parameters, whereas b) shows the final alignment result. The 2D image from Fig. 6.6 on page 74 is seen (Cropping and colormap has been changed, and a lower threshold for intensity values been set. Lower intensities are pictured in grey). The blue dots indicates the markers on the 2D image. The 3D geometric grid is sketched with the red frame, representing the edges of the profile, and the red dots representing the 3D positions of the markers. The yellow dots represents the projection of the 3D markers onto the 2D image. The aligning procedure can be visualized as a translating and rotating the 3D geometry and changing camera parameters until the yellow dots cover the blue. The bars at the right side of each figure represents the error norm for the alignment. Because of the intensity threshold, some of the profile at the far end of the profile and at the leading edge, where the intensities were very low, is colored grey. The apparent gap between the 2D image and the projected 3D grid is thus just a feature of the colormap.
calibration function. Even when conducting the in situ calibration, there can still be a non-negligible discrepancy between the pressure measured by reference pressure sensors and PSP. The discrepancy is in form of a small offset in pressure value. This is a well-known problem from steady PSP where this offset often occurs, but is not yet understood. This is the reason why PSP, even though it is principally an absolute pressure sensor, is always used in combination with at least one reference pressure sensor, whose measurements are then used to perform the offset correction. In the present case a subset of the kulite sensors were used for this correction. The kulites were chosen in locations where the PSP measurements were trusted (i.e. not close to the edges), and where the pressure gradient was small. For these positions the mean deviation between kulite and PSP data was found and used as a global pressure offset.

6.3.5. Result

The final instantaneous pressure field for this measurement point is shown on Fig. 6.9. The main feature of the pressure field, is two shocks along the span, recognized as distinct low-to-high pressure transitions. The existence of dual shocks are known for this profile at steady flow away from design conditions [28] and in the unsteady case, moving dual shocks have been numerically investigated in [32] at similar flow conditions. At the leading edge the optical access was bad, and furthermore the paint was partly destroyed due to impacting particles for previous measurements, as was seen on Fig. 6.5 on page 73. The three-dimensionality of the flow due to wall influence is clearly seen, resulting in bending of the shock lines, and high local spanwise pressure
6.4 Unsteady PSP correction

The pressure values from the reference and the iPSP section respectively is plotted as function of $t/T$ and $x/c$ on Fig. 6.10. This provides an overview of the dynamic pressure distribution in the respective sections within one oscillation period. The main difference is that the PSP data is available over the whole $x/c$ range, and the higher spatial resolution. The pressure distributions are seen to be very equal in shape and magnitude. From Fig. 6.10 the course of events within one period can be interpreted: At $t/T = 0$ there are two distinct shocks at $x/c \approx 0.2$ and $x/c \approx 0.5$. As time progresses, the first shock moves aft, the latter remaining largely stationary, until the two merge to one at $t/T \approx 0.4$. The shocks separate again at $t/T \approx 0.6$, and the first shock then starts moving back towards the leading edge. The iPSP data is interpolated onto the reference pressure sensor spatial positions, and compared for $t/T = 0, 0.25, 0.50$ and $0.75$ on Fig. 6.11 on the next page. The shape of the pressure curves agree quite well, however at the moving the paint is not able follow the shape and amplitude. For example for $t/T = 0.75$ where the shock is moving towards lower $x/c$, the PSP pressure is clearly lagging compared to the kulite measurements. For a

![Figure 6.10](image-url)
6.4 Unsteady PSP correction

qualitative investigation, however, the uncorrected results are sufficiently accurate. If

![Graphs showing PSP pressures at different times](image)

**Figure 6.11.**: PSP (dots) and reference pressures (circles) for selected $t/T$. The greater spatial resolution and range of iPSP is seen. For $t/T = 0$ where the leading edge shock is moving relatively slow (Compare with Fig. 6.9 on page 79) the iPSP pressure resembles the reference well, while for $t/T = 0.75$ where the shock moves rapidly, the iPSP cannot keep up.

on Fig. 6.10 on the preceding page a fixed $x/c$ is chosen, the corresponding time series is obtained. On Fig. 6.12 on the next page examples are seen for four chordal positions.

### 6.4.1. Frequency analysis of the measured signal

To correct the unsteady PSP pressures, the time series for each spatial point must be transformed into the frequency domain, for multiplication with the transfer function. Since the correction can only be applied for frequencies where the transfer function is
Figure 6.12: \( \text{PSP (dots) and reference pressures (circles) for selected } x/c. \) For \( x/c = 0.050 \) the pressure variations are slow and small, and the iPSP follows the reference pressure. For \( x/c = 0.050 \) large pressure jumps appear when the shock passes, and the PSP signal lags and has reduced amplitude. For \( x/c = 0.533 \) and \( x/c = 0.683 \) the pressure remains relatively constant.
measured, i.e. up to 120 Hz, the frequency domain representation of the pressure should be represented with frequencies in the same range. Since the number of points per period, \( N_{pp} = \frac{f_{sc}}{f_s} \), is an integer, the pressure signal, \( P_n \), can be represented exactly by using the discrete Fourier transforms for real input (See App. A.4 on page 100):

\[
P_n = \frac{1}{N_{pp}} \sum_{k=0}^{N_{pp}/2} X_k e^{i \frac{2\pi k}{N_{pp}} n} \quad \text{where} \quad X_k = \sum_{n=0}^{N_{pp}-1} P_n e^{-i \frac{2\pi k}{N_{pp}} n}
\]  

(6.12)

where the times corresponding to \( n \) are \( t_n = \frac{n}{f_s} \), and the frequencies corresponding to \( k \) are \( f_k = k f_s f_{sc} \). The value for \( k = 0 \) is the mean value of the signal. Taking the first \( j \) terms of (6.12), yields an approximation to \( P_n \)

\[
P_n^{(j)} = \frac{1}{N_{pp}} \sum_{k=0}^{j} X_k e^{i \frac{2\pi k}{N_{pp}} n}
\]

(6.13)

The time series are all approximated with \( P_n^{(j)} \), such that the highest frequency component is for \( f = 120 \) Hz. An alternative to the DFT is to use the previously mentioned LS fitting algorithm, for determination of the signal frequency content. This is a much more general method which also works for aperiodic and noisy signals, as well as signals which are not sampled optimally. On Fig. 6.13 on the next page approximation of the measured signal with four sines have been carried out for the two methods at two different chordal sections.

It is seen that the two methods produce virtually identical results. It is seen that the time series have very different spectral content. On Fig. 6.13a, the curve is quite smooth, and the approximation is good. On Fig. 6.13b, the shock is traveling across the section twice per period, introducing strong high-frequency components into the signal, why four sines are insufficient. The number of DFT terms required to resolve the sampled signal to a given accuracy, can be determined. The local relative error of the fit is defined as

\[
\epsilon^{(j)}(x,t) = \frac{P_n^{(j)}(x,t) - P_{\text{meas}}(x,t)}{P_{\text{meas}}(x,t)}
\]

(6.14)

For a given \( j \), for each time series, the maximum and the average of the absolute value of the error is found, \(|\epsilon_{\text{max}}|(x)\) and \(|\bar{\epsilon}|(x)\), respectively. These quantities provides a measure for the quality of the fit as function of \( x \). Taking the spatial average and maximum of these values yields global estimates for the error, summarized in Tab. 6.3 on the following page, and plotted on Fig. 6.14 on page 85, as function of the number of sine terms in the fit. It is seen that all error estimates goes to zero for 18 terms, where the sampled signal is exactly represented. The easiest norm to interpret is \( \max(|\epsilon_{\text{max}}|) \), which is the maximal global error of the fit. The error decreases dramatically by including up to three terms, then stagnates up to nine terms. This reflects the fact that the signal is quite well resolved, except for the shock, which contains so high frequencies that including the terms 4-9 does not effectively resolve it.
Figure 6.13.: Approximating pressure time with four sine functions, for the DFT, $P_{DFT}^{(4)}$, and LS method $P_{LS}^{(4)}$. For $x/c = 0.355$ the signal shape is quite well approximated, while for $x/c = 0.464$, the passing shock causes abrupt pressure changes which cannot be represented with sines of low frequency.

| $\|\epsilon\|_{\text{max, max}}$ | $\max(\|\epsilon\|_{\text{max}})$ | Maximal global error |
| $\|\epsilon\|_{\text{max, av}}$ | $\max(\|\epsilon\|)$ | Maximal error of time-averages |
| $\|\epsilon\|_{\text{av, max}}$ | $< \|\epsilon\|_{\text{max}}>$ | Average of maximal time series error |
| $\|\epsilon\|_{\text{av, av}}$ | $< \|\epsilon\| >$ | Average error of time-averages |

Table 6.3.: Error estimates for sinusoidal fits
Figure 6.14: Error norms for sinusoidal fit as function of number of sine terms.
better, than using only the first three. On the other hand the average of the maximal error for each time series, \( \langle |\varepsilon|_{\text{max}} \rangle \), drops monotonically, showing that the error on other spatial positions becomes smaller. If instead of the maximal error for each time series, the time-average is considered, it can also be represented by its maximum and mean value, \( \max(|\varepsilon|) \) and \( \langle |\varepsilon| \rangle \), in the same fashion. This is seen to produce smaller error estimates, but with the same trend: The overall error decreases steadily, but at the shock, a large number of sine terms are required to describe the local behavior. From Fig. 6.14 on the preceding page the maximal error introduced fitting the PSP signal with four sines, can be seen to be 6 percent, whereas the typical error is considerably smaller.

### 6.4.2. Amplitude and phase correction

Given that the pressure variation is adequately represented by \( P^{(4)} \), each term can be corrected with the transfer function of the paint, \( H_{\text{paint}}(\omega) \). This is simply done by correcting the Fourier coefficients with the transfer function of the paint, \( H_{\text{paint}} \), in (6.13). The corrected pressure, \( P_{\text{corr}}^{(j)} \), become

\[
P_{\text{corr}, n}^{(j)} = \frac{1}{N_{pp}} \sum_{k=0}^{j} \frac{X_k}{H_{\text{paint}}(f_k)} e^{i \frac{2\pi n}{N_{pp}}} n
\]

or as a real quantity in terms of gain, \( g(f) \) and phase shift \( \Delta \phi(f) \):

\[
P_{\text{corr}, n}^{(j)} = \frac{1}{N_{pp}} \sum_{k=0}^{j} \frac{|X_k|}{g(f_k)} \cos \left( \frac{2\pi n}{N_{pp}} + \arg(X_k) - \Delta \phi(f_k) \right)
\]

On Fig. 6.15 on the next page the reference and PSP pressures for \( x/c = 0.467 \), and their respective fits are shown. Two different transfer functions are used to illustrate the correction, namely that of the original iPSP probe from Sec. 5.1 on page 46 and that of the re-synthesized iPSP (with new FIB) from Sec. 5.2 on page 47. Several things are seen from Fig. 6.15 on the following page. First of all the corrections are seen to have the right qualitative effect, namely producing a faster reacting signal with higher amplitudes, but it is also seen that the effect of the correction is negligible for the original iPSP transfer function. For the considerably slower re-synthesized iPSP, the effect of the correction is more marked, as the shape of the corrected pressure curve is clearly closer the fitted reference pressure curve, than the uncorrected curve. If one disregards the offset error (which is present in the measured signal, influenced by the offset correction, but unaffected by the unsteady correction) the curve shape is even very similar. A way of comparing the shape of the corrected PSP pressure \( P_{\text{corr}}^{(4)} \) with the fitted reference pressure, \( P_{\text{ref}}^{(4)} \), is to compute the correlation coefficient.
Figure 6.15: Unsteady correction for $x/c = 0.467$. The PSP and Kulite data have been fitted with 4 DFT terms. The "fast paint" has the transfer function measured for the iPSP in Sec. 5.1 on page 46, and the "slow paint" is fitted from the resynthesized iPSP (new FIB) from Sec. 5.2 on page 47, which proved to be significantly slower than the original iPSP.
6.4 Unsteady PSP correction

This is computed for the uncorrected fitted pressure, $P_{(1)}^{(4)}$, and for the pressure corrected with the two transfer functions $P_{corr, fast}^{(4)}$ and $P_{corr, slow}^{(4)}$ respectively and shown on Fig. 6.16. Along the first half of the chord, in the region where the shock travels, the pressure amplitudes are relatively high, and the discrepancy between reference and PSP pressure very localized at the shock positions, giving a high correlation coefficient. Along the aft half of the chord, the pressure amplitudes are very small, and much more sensitive to noise, thus the lower correlation in this region. In the transition between these two regions, where the shocks merge and split, the shape of the pressure is more complicated and not well represented by the fitted curve. The error caused by phase shift thus becomes large. This is especially the case at $x/c = 0.425$ and $x/c = 0.517$. In the region of interest, where the pressure changes are significant, $0.2 < x/c < 0.6$, it is seen that the fast paint correction increases the correlation coefficient, and the slow

\[
R(P, P_{ref}^{(4)}) = \frac{(P - \overline{P})(P_{ref}^{(4)} - \overline{P}_{ref}^{(4)})}{(P - \overline{P})^2 (P_{ref}^{(4)} - \overline{P}_{ref}^{(4)})^2}
\]

(6.17)

**Figure 6.16.** Correlation coefficient between $P^{(4)}$ and $P_{corr}^{(4)}$ for the two paints. It is seen that the correction improves the correlation.
paint correction even more. This does not prove that the slow paint transfer function is the true, even though it is made plausible that the applied paint reacts a somewhat slower than the original iPSP probe. This is however not the only explanation. A "reverse engineering" approach was tested, to see if the paint transfer function could be deduced from the measurements. The amplitudes and phases for all $P^{(4)}$ and $P_{\text{ref}}^{(4)}$, were compared directly, and the conclusion was that there were no unique transfer function which could provide a reasonable description for the correction of all time series simultaneously. The best overall choice however were quite close to seems to the transfer function of the slow paint. On figure 6.17a, a plot of the relative pressure error, $\epsilon^{(4)}$

$$\epsilon^{(4)} = \frac{P^{(4)} - P^{(0)} - \left( P_{\text{ref}}^{(4)} - P_{\text{ref}}^{(0)} \right)}{P_{\text{ref}}^{(4)}}$$

is shown. On this type of plot amplitude discrepancies will appear mainly as the size of the error while phase discrepancies will mainly affect the homogeneity of the plot. That there is a phase shift between the signals is seen from the fact that the error is quite asymmetric, concentrated along the path of the traveling shock, especially when this is traveling toward the leading edge (compare with Fig. 6.10 on page 80). It shows that the frequency components of the two signals are misaligned. The error is largest at the shock position, especially for the shock traveling toward the leading edge. In the suction region, between the leading edge and the shock, the error shows signs of being periodic, a clear sign of a phase shift between the two signals. The effect of correcting with the fast paint is as expected very small. On Fig. 6.17b the effect of the correction with the slow paint transfer function is seen. It is seen that the error at the shock is reduced significantly, while reducing some of the suction region variation are evened out. All in all the effect of the correction is a clear but insufficient improvement. In Tab. 6.4 the mean and maximal error is shown for the "unsteady region", $0.2 \leq x/c \leq 0.6$, to cut out the boundary regions with large errors and regions, where the pressure is practically steady. The first row shows error between raw PSP and raw reference pressures, and the second the error when representing the PSP data

| $a$ | $b$ | $\max(|a - b|)/P_{\text{ref}}$ | $\text{mean}(|a - b|)/P_{\text{ref}}$ |
|-----|-----|------------------|------------------|
| $P$ | $P_{\text{ref}}$ | 17.66% | 2.78% |
| $P^{(4)}$ | $P_{\text{ref}}$ | 18.33% | 2.87% |
| $P_{\text{corr,fast}}^{(4)}$ | $P_{\text{ref}}$ | 17.22% | 2.83% |
| $P_{\text{corr,slow}}^{(4)}$ | $P_{\text{ref}}$ | 15.43% | 2.92% |
| $P^{(4)} - P^{(0)}$ | $P_{\text{ref}}^{(4)} - P_{\text{ref}}^{(0)}$ | 8.85% | 1.95% |
| $P_{\text{corr,fast}}^{(4)} - P_{\text{corr,fast}}^{(0)}$ | $P_{\text{ref}}^{(4)} - P_{\text{ref}}^{(0)}$ | 8.24% | 1.79% |
| $P_{\text{corr,slow}}^{(4)} - P_{\text{corr,slow}}^{(0)}$ | $P_{\text{ref}}^{(4)} - P_{\text{ref}}^{(0)}$ | 8.07% | 1.57% |

Table 6.4.: Global error norms for various raw and fitted pressure values
by the its 4-terms sinusoidal fit. The error is slightly increased, and even when applying the corrections in row three and four, the size of the error remains relatively unchanged. It is concluded, that considering the initial offset between the reference and PSP signal, the error committed during fitting and the relatively small effect of correction, the quality of the PSP data is not enhanced by correction, when comparing to the raw reference pressures. However when the reference pressures are represented with the same type of fit as the PSP data, and only the oscillating part of the pressures are considered (as in Fig. 6.17), the effect of the correction becomes apparent, as seen in row 5-7 of Tab. 6.4 on the previous page.

![Figure 6.17: Error norms for measured and fitted PSP and reference pressures](image)

### 6.5. Discussion and Summary of Wind Tunnel Results

The experiment in general was successful. The unsteady measurement setup and data acquisition worked as expected, and with the current hardware a flash duration of 200 $\mu$s provided sufficient excitation light for acquisition of a fully analyzable PSP image. Therefore the technique is truly unsteady in the sense that no phase-domain averaging was is needed to acquire the single images, that each raw image is therefore an instantaneous image, and that the image acquisition thus needs no assumption of periodicity of the flow. In practice, however, the sampling rate is limited to 0.2 Hz by the recharging time of the excitation light source, and to attain a higher time resolution for measurements, the studied flow must be periodic or repeatable to with high accuracy. The iPSP performed overall satisfactory, except for it being too easily destroyed by impacting particles. This would be solved by lowering the amount of
filler, but this would increase the response time, so it seems that the current iPSP formulation cannot be improved dramatically without either sacrificing the fast response time or durability. The iPSP provided, facilitated by ToPas, good online visualization. The used analysis approach relies on several facts, first of all it requires periodicity of the measured signal. For most unsteady applications the investigated flow will be periodic, e.g. in cases of an oscillating profile or a rotating propeller or similar, but in some cases it is a transient phenomenon that is of interest. If the flow is not periodic, the DFT cannot be used, and LS fitting of the transient with sines must be done instead. The periodicity of the flow could in this case be checked directly, from the similarity of the four images and the four sets of reference pressures used for phase-resolved averaging. In the present case the correspondence between values from each four periods was excellent, only small local deviations from periodicity appeared at the position of the shock. Factors which can destroy the periodicity is random noise on the signal (can be removed by sufficient phase-resolved averaging), and imperfect sampling.

The investigation showed that the iPSP was able to determine unsteady pressure distributions with errors in the order of few percent after offset correction, with the largest errors at the location of the fastest pressure changes. The main problem problem of this investigation was that the nature of the measured pressure variations were not really suitable for validation of the correction method, as long as the transfer function for the paint is only known for frequencies up to 120 Hz. Firstly, a shock passing a point on the surface results in a step function-like time variation of the pressure in that point. In other words the frequency content at high frequencies is not negligible. This means that to properly represent the physical signal with the sampled, the sampling frequency must be high as a consequence of the sampling theorem (A.11). To subsequently represent the sampled signal with a limited number of Fourier expansion terms, is in the case of traveling shocks an approximation which is not very good, and limits the validity of the analysis. It is of course important to stress that all of this is only a problem because the transfer function for the paint is unavailable at high frequencies.

How fast pressure changes the PSP can measure is therefore depending on the frequency content of the signal. As of now iPSP can accurately correct frequencies up to 120 Hz. This means that if a signal is close to sinusoidal, it can have a frequency of up to 120 Hz and still be corrected with high precision. If in contrast 12 terms of a Fourier expansion is required to represent a signal properly, the oscillation frequency of the signal cannot exceed 10 Hz. The limit of 120 Hz can probably be raised to 150 Hz or even 180 Hz by further optimization of the current setup, at the expense of significantly larger uncertainties on the transfer function, but the limit cannot be dramatically extended.

In the present signal the assumption of linearity is also subject to discussion. Typically the signal has a base pressure of 25-30 kPa and the amplitude of the largest harmonic components is in the order of 3 kPa. This corresponds to $P_0^* = 0.25 - 0.30$ and $P_1^* = 0.03$ on Fig. 4.5b, where the maximal error can be read off to be approximately
10 %, which is still acceptable. It must be concluded that the measured PSP signals can only for some chordal sections be described sufficiently accurate with sines of up to 120 Hz. For some sections the traveling shocks will make the representation of the signal inaccurate and introduce larger errors than the correction will eventually remedy. For other sections pressure variations are so small, that it is not necessary to correct phase and amplitude at all (See e.g. Fig. 6.12 on page 82).
7. Conclusion

A new experimental setup for characterization was characterized and used to conduct parametric investigations of the frequency response of unsteady pressure sensitive paints. The calibration of PSP was used for correction unsteady pressures measured on an oscillating airfoil.

The experimental setup was presented and the optical detection system designed and optimized for characterization of paints for frequencies up to 120 Hz and with modulation amplitudes of up to 3 kPa. It was shown that the complete measurement system was sufficiently linear and time invariant, to be described with a transfer function. It was shown that the frequency response was partly due to a capacitive effect in the light detection circuit, but that the transfer function at lower frequencies was dominated by other effects. The transfer function was determined for relevant light detecting circuits to sub-percent accuracy, and used for correction of the paint measurements. The setup and the measurement methodology is now readily available for testing and calibration of future unsteady PSP formulations.

The parametric investigations for variations of the iPSP formulation, yielded the following results: The response time depends on the thickness of the samples, roughly follows the power law for diffusive paint layers, and that paint layers above a certain thickness seems to be insensitive to further thickness changes. It was found that the addition of TiO$_2$ to the paint systematically enhanced the frequency response, only changing the characteristic time scale, while on the other hand making the paint more brittle. Addition of reference component provided no significant effect in itself but in combination with TiO$_2$ it yielded fast-responding with an upper frequency of well above 100 Hz. The re-synthesis of this formulation was unsuccessful, and it was concluded that neither aging of the polymer nor ultrasound treatment in the synthesis process was sufficient explanation for the difference.

The iPSP was used for measurement of phase-locked full-field pressure distributions on the suction side of an oscillating profile in transonic flow. Under assumption of periodicity, the phase-locked data were recombined to give an equivalent sampling rate of 3840Hz with full spatial resolution. This gave an excellent visualization of the dynamics of the pressure distribution, which can neither be obtained by conventional pressure measurement techniques, nor conventional PSP. The iPSP generally agreed well with reference pressure measurements, sufficiently well for qualitative conclusions to be made based on uncorrected measurements. The local offset error were typically larger than the error due to the frequency response of the paint. The shocks introduced significant high-frequency components into the pressure variations, and the
results could generally not be represented appropriately within the frequency range of the transfer function. The analysis thus introduced errors of the same order of magnitude as the potential benefit from the correction, but it is expected the correction will yield good results for smoother pressure variations, or by extending frequency range of the transfer function. As a part of the continuation work, the iPSP correction scheme is currently being investigated in connection with square and triangular pressure variations.

Unsteady PSP for industrial applications has a vast potential, but still many challenges must be overcome. In this work the problems of developing a formulation which is more durable and less sensitive to minor variations in composition and synthesis procedure are pointed out. Other paints, other bottlenecks. Because of the and the many different requirements to an industrial applicable iPSP’s, a step forward in one respect will more often than the opposite, be a step backwards in another.

With the unsteady PSP project of DLR Göttingen considerable advancements has been made for both paint synthesis, and within development of measurement systems and software. Combined with the fact that the rapid development of new cameras, light detectors and sources continually widens the envelope of applicable PSP formulations and measurement techniques, there is reason form optimism for the future of unsteady pressure sensitive paints.
Bibliography


A. Signal processing theory

In this appendix the necessary signal processing theory will be briefly stated. The chapter is based on standard formulas from mainly [18, 33, 34]. It has been attempted to make the chapter relatively complete to understand the application in the report. The most basic and classical theorems have simply been stated, while the central results have been derived.

A.1. LTI system

Consider a system

\[ y(t) = f(x(t)) \]  \hspace{1cm} (A.1)

where \( x(t) \) is the input signal and \( y(t) \) is the output signal and \( f \) is an operator defining relation between input and output. If \( f \) is a \textit{Linear, Time-Independent} operator the system has some very nice properties. A \textit{linear} operator, \( f \), by definition, possess the properties of superposition and homogeneity. An input signal \( x(t) \) can in the general case be thought as the sum of \( n \) signal components, i.e. \( x(t) = \sum_n x_n(t) \).

In that case superposition requires that

\[ f \left( \sum_n x_n(t) \right) = \sum_n f(x_n(t)) \]  \hspace{1cm} (A.2)

while homogeneity that

\[ f(k \cdot x(t)) = k \cdot f(x(t)) \]  \hspace{1cm} (A.3)

Written together this gives that if \( f \) is linear then

\[ f \left( \sum_n k_n x_n(t) \right) = \sum_n k_n f(x_n(t)) \]

A \textit{time-independent} operator is one that is not explicitly depending on time, i.e.

\[ y(t + \tau) = f(x(t + \tau)) \quad \forall \quad \tau \in \mathbb{R} \]  \hspace{1cm} (A.4)

For instance \( y(t) = g(x(t)) \) is time independent while \( y(t) = g(x(t), t) \).
Consider now the input to the signal, \( x(t) \). As any function it can be written as a superposition of delta functions:

\[
x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau
\]

(A.5)

by the definition of the Dirac delta function. The above integral can be recognized as \((x \otimes \delta)(t)\). This can be seen from the following proof. Now use this as the input function to the system

\[
y(t) = f \left( \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau \right)
\]

(A.6)

\[
= \int_{-\infty}^{\infty} f [x(\tau)\delta(t - \tau)] d\tau \quad (f \text{ is linear})
\]

\[
= \int_{-\infty}^{\infty} x(\tau)f[\delta(t - \tau)] d\tau \quad (x(\tau) \text{ is constant in } t, f \text{ is linear})
\]

\[
= \int_{-\infty}^{\infty} x(\tau)h(t - \tau) d\tau = (x \otimes h)(t)
\]

Therefore an LTI system can be completely characterized by the function

\[
h(t) = f(\delta(t))
\]

(A.7)

which is called the impulse response. The impulse response is the system response to an impulse of finite energy but infinitely short duration, which is of course an idealization, while still an extremely useful concept. By applying the convolution theorem (A.21) to (A.7), it appears that

\[
\mathcal{F} [y] = Y(\omega) = \mathcal{F} [(x \otimes h)(t)]
\]

(A.8)

\[
= \mathcal{F} [(h \otimes x)(t)] \quad \text{(convolution commutes)}
\]

\[
= H(\omega)X(\omega)
\]

This is identified as the transfer function of \( f \). To sum up: An LTI system is fully characterized in the time domain by its impulse response \( h(t) \) and in the frequency domain by its transfer function \( H(\omega) \). \( H(\omega) = \mathcal{F} [h(t)] \) and \( h(t) = \mathcal{F}^{-1} [H(\omega)] \).

### A.2. Sampling

A Dirac comb, \( \Delta_T(t) \) is the function

\[
\Delta_T(t) \equiv \sum_{n=-\infty}^{\infty} \delta(t - nT)
\]

(A.9)
where $\delta$ is the Dirac delta function. The Dirac comb is the sum of a train of equally spaced delta function, giving the graphical representation that earns the function its name. Consider a continuous-time signal $x(t)$. Sampling this signal with sampling frequency $f_s = 1/T$ where $T$ is the sampling time, gives a sampled signal $x_T(t) = x(nT)$ where $n \in \mathbb{Z}$. This is called an ideal sampling, and can be mathematically expressed by multiplication with (A.9):

$$x_T(t) = \sum_{n=-\infty}^{\infty} x(nT) \delta(t - nT)$$  \hspace{1cm} \text{(A.10)}$$

It is clear that this expression is zero everywhere, unless for $t = nT$ where it takes the value $x(nT)$. This is a way of representing a sampled signal with a continuous function.

### A.3. Nyquist-Shannon Sampling Theorem

A bandlimited signal is one that contains no energy at frequencies higher than the bandwidth, $B$. To be able to exactly reproduce continuous signal from the samples it must be true that

$$\frac{f_s}{2} > B \hspace{1cm} \text{(A.11)}$$

This fundamental theorem in signal processing was published in [34]. $\frac{f_s}{2}$ is also called the Nyquist frequency. If one wants to represent a continuous signal unambiguously, the signal must contain no energy above the Nyquist frequency. If this is not the case, frequency components of higher frequency will be irrevocably indistinguishable from lower frequencies (aliasing). If the signal is not already band limited, it must be lowpass filtered or sampled at a higher rate, such that (A.11) is fulfilled.

### A.4. Transforms

The continuous Fourier transform is

$$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt$$ \hspace{1cm} \text{(A.12)}$$

(For convenience and tradition the imaginary unit is not included as part of the argument). Now apply this continuous Fourier transform to a discrete signal, i.e. apply
(A.12) to (A.10). This yields

\[ X(\omega) = \int_{-\infty}^{\infty} \left( \sum_{n=-\infty}^{\infty} x(nT) \delta(t - nT) \right) e^{-i\omega t} dt \]  

\[ = \sum_{n=-\infty}^{\infty} x(nT) \left( \int_{-\infty}^{\infty} \delta(t - nT) e^{-i\omega t} dt \right) \]

\[ = \sum_{n=-\infty}^{\infty} x(nT) e^{-i\omega nT} \]  

(A.13)

With the substitution \( x_n = x(nT) \), and evaluated for a finite length signal, \( n = 0, 1, \ldots, L \), (A.14) becomes

\[ X(\omega) = \sum_{n=0}^{L-1} x_n e^{-i\omega n} \]  

(A.14)

which is known as the discrete-time Fourier Transform (DTFT). Evaluation the transform for \( N \) equidistant \( \omega \)-values within one period of such that

\[ \omega_k = \frac{2\pi k}{N} \]  

where \( k = 0, 1, \ldots, N - 1 \), it becomes

\[ X(\omega_k) = \sum_{n=0}^{L-1} x_n e^{-i \frac{2\pi k}{N} n} \]  

(A.16)

If the sampled signal corresponds to one period of the signal, then \( N = L \), such that

\[ X(\omega_k) = \sum_{n=0}^{N-1} x_n e^{-i \frac{2\pi k}{N} n} \]  

(A.17)

which is known as the discrete Fourier Transform (DFT). The inverse of this transform, the IDFT, is

\[ x_n = \frac{1}{N} \sum_{k=0}^{N-1} X(\omega_k) e^{i \frac{2\pi k}{N} n} \]  

(A.18)

In the case where the \( x_n \) are real it can be shown that \( X(\omega_k) = \overline{X(\omega_{N-k})} \), where the overline denotes complex conjugation. By meas of this the IDFT can be reduced to

\[ x_n = \frac{1}{N} \sum_{k=0}^{N/2} X(\omega_k) e^{i \frac{2\pi k}{N} n} \]  

(A.19)

in the case where \( N \) is even. The element corresponding to \( k = 0 \) in these series is related to a frequency of zero, i.e. it gives the mean level of \( x_n \).
A.5. Convolution

The convolution of a signal $x(t)$ with a kernel $h(t)$ is denoted $(h \otimes x)(t)$ is defined as

\[
(h \otimes x)(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau
\] (A.20)

The convolution theorem for Fourier transforms states that convolution in the time domain corresponds to a multiplication in the frequency domain and vice versa.

\[
\mathcal{F}[(h \otimes x)(t)] = \mathcal{F}[h(t)] \mathcal{F}[x(t)]
\] (A.21)

A.6. Transfer function

Consider a system with a continuous input signal $x(t)$ and output signal $y(t)$. If the system is linear a transfer function exist if the system is linear and time-invariant, The transfer function describes the relationship between the input and the output signal in the frequency domain:

\[
H(s) = \frac{\mathcal{F}[y(t)]}{\mathcal{F}[x(t)]} = \frac{Y(\omega)}{X(\omega)} = \frac{\int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt}{\int_{-\infty}^{\infty} y(t)e^{-i\omega t}dt}
\] (A.22)

In the frequency domain, the description of the system is

\[
Y(\omega) = H(\omega)X(\omega)
\] (A.23)

An important consequence of this is that a LTI system does only affect the complex amplitude of a harmonic signal. If the input signal is a pure harmonic signal, i.e.

\[
x(t) = Xe^{i\omega t} = |X|e^{i(\omega t + \arg(X))}
\] (A.24)

Then

\[
Y(\omega) = H(\omega)X(\omega)
\]

(A.25)

\[
= |H(\omega)|e^{i\arg(H(\omega))} \cdot |X(\omega)|e^{i\arg(X(\omega))}
\]

\[
= (|H(\omega)| \cdot |X(\omega)|)e^{i(\arg(H(\omega))+\arg(X(\omega)))}
\]

and thus

\[
y(t) = (|H(\omega)| \cdot |X(\omega)|) e^{i(\omega t + \arg(H(\omega))+\arg(X(\omega)))}
\] (A.26)

It is seen that amplitude and phase are potentially changed by the system, whereas the frequency is left unchanged. to express the influence of the transfer function in real quantities, the gain and the phase shift is introduced. The gain is defined as

\[
g(\omega) = \lim_{\omega \to 0} \frac{|H(\omega)|}{|H(\omega')|}
\] (A.27)
and the phase shift
\[ \Delta \phi(\omega) = \arg(H(\omega)) \]  
(A.28)

The (normalized) transfer function can be written then be written in the form
\[ H(\omega) = g(\omega)e^{i\Delta \phi(\omega)} \]  
(A.29)

### A.7. First-order system

A first-order system is an dynamical system which is governed by the differential equation
\[ a_1 \partial_t y + a_0 y = b_0 x \]  
(A.30)

where \( x \) ist the input and \( y \) is the output. This can also be written
\[ (\tau \partial_t + 1)y = Kx \]  
(A.31)

where \( \tau = \frac{a_1}{a_0} \) is the time constant and \( K = \frac{b_0}{a_0} \) is the static amplification. The ratio between input and output is
\[ \frac{y}{x} = \frac{K}{\tau \partial_t + 1} \]  
(A.32)

Taking the Fourier transform \( \mathcal{F} \) on either side of this equation yields exactly the transfer function
\[ H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{i\omega \tau + 1} \]  
(A.33)

where the static amplification has been removed, by normalizing the transfer function to one at \( \omega = 0 \). The gain and phase shift can be identified as
\[ g(\omega) = |H(\omega)| = \frac{1}{\sqrt{1 + (\omega \tau)^2}} \quad \text{and} \quad \Delta \phi(\omega) = \arg(H(\omega)) = \arctan(\omega \tau) \]  
(A.34)

To see how a first order system responds to a step, consider the step function
\[ x(t) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases} \]  
(A.35)

whose Laplace transform is \( \mathcal{L}[x(t)] = \frac{1}{s} \). In the frequency domain the convolution \( h(x) \otimes x(t) \) corresponds to the multiplication \( H(s)X(s) \):
\[ Y(s) = \frac{1}{s} \frac{1}{1 + s \tau} = \frac{1}{s(\frac{1}{\tau} + s)} \]  
(A.36)
To get to the back to the time-domain, the inverse Laplace is used

\[ y(t) = \mathcal{L}^{-1} \left[ \frac{1}{\tau s(\frac{1}{\tau} + s)} \right] = 1 - e^{-\frac{t}{\tau}} \]  

(A.37)

The used Laplace transforms are tabulated in various mathematical handbooks as e.g. [33]. The upper frequency of a first order system is found setting \( g \) equal 0.9 in (A.34) and solving for \( f_u \)

\[ f_u = \sqrt{\left(\frac{0.9}{2\pi}\right)^2 - 1} \frac{1}{\tau} \approx \frac{0.0771}{\tau} \]  

(A.38)

### A.8. Polar representation of a transfer function

The polar graph shows the transfer function as a graph in the complex plane. It is simply shows the complex values of the (normalized) transfer function, or to put it otherwise, it shows a polar plot with \( g \) as the modulus and \( \Delta \phi \) as the argument. Since it shows only the relationship between gain and phase shift, the frequency information is not present in this plot, and for this reason transfer functions which are similar with respect to a characteristic time will turn out identically in this plot. Bear in mind that the reverse is not necessarily the case, but only the prerequisite for similarity with respect to a time scale. On Fig. A.8 on the next page a selection of different transfer functions are seen in the polar representation. Consider the graph for the first-order system with the transfer function (2.37). As all the transfer functions, it starts in \((\rho, \theta) = (1,0)\), and following the graph for increasing frequency the gain becomes smaller and the phase shift becomes larger. The graph follows a circular arc which ends at the origin. This defines the asymptotic behaviour for \( \omega \to \infty \). In this case \( g(\omega) \to 0 \) and \( \Delta \phi(\omega) \to -90^\circ \). The first order system is a special case of the lag system, (2.38). A lag system is as seen represented by a circular arc, which does not go through the origin. This means that the gain converges to the value where the graph hits the abscissa, and the phase shift converges to zero. On the figure also the transfer function for the PSP diffusion model is seen. The gain is seen to converge to zero and the phase shift to \(-45^\circ\). Two cases of the DOT model is seen. It is seen that the phase shift becomes almost constant for a while, then starts to change again, eventually converging to zero. For small optical thickness, the phase shift is also small, while for increasing optical thickness, it converges to the graph of the PSP diffusion model. Lastly an ideally responding system is simply represented by the point \((\rho, \theta) = (1,0)\).
Figure A.1: Polar representation of various systems

- **Red** DOT, \( \sigma^* = 0.1 
- **Blue** DOT, \( \sigma^* = 0.5 
- **Black** Pure diffusion [6]
- **Green** Lag model, \( \tau_1/\tau_2 = 0.50 
- **Pink** Lag model, \( \tau_1/\tau_2 = 0.25 
- **Light Blue** First order system
B. The General Minimization Problem

This section briefly lays down the theory on which the sine-fitting procedure and the 3D-alignments procedure is based. This review is partially a summary of the theory from [17] accompanied by some concrete examples for the present application.

Let $t = \{t_0, t_1, ..., t_{N-1}\}$ denote discrete times, and $h(t)$ corresponding values of a signal. Let $f(t, x)$ where denote an approximating analytic expression with $x = \{x_1, x_2, ..., x_k\}^T$ being a vector with $k$ elements. These elements are the free parameters in the approximating expression. Now let $g(x)$ be the functional

$$g(x) = \|h(t) - f(t, x)\|$$  \hspace{1cm} (B.1)

where the norm $\|\|$ is any desired vector norm. The functional $g(x)$ thus represents the error of the approximation, and thus the goal is to minimize $g(x)$, i.e. to determine

$$x \mid g(x) = \min(g(x))$$  \hspace{1cm} (B.2)

Most commonly used as norm, is the two-norm, in which case (B.1) defines the Method of Least Squares. In general the problem is nonlinear and have no closed-form solution. A necessary condition for $g(x)$ to attain its minimal value is that

$$\partial_{x_i} g(x) = 0$$  \hspace{1cm} (B.3)

where $g(x)$ is possibly not differentiable in a strict mathematical sense. Using the Euler method with a step size of $\Delta x$ the gradient can be numerically approximated:

$$\partial_{x_i} g(x) \approx \frac{g(x + e_{x_i} \Delta x) - g(x)}{\Delta x}$$  \hspace{1cm} (B.4)

where $e_{x_i}$ is the unit vector for the $i^{th}$ coordinate.

B.0.1. Method of negative gradients

Given initial guess of $x^0$ and step size $\Delta x$, where the superscript denotes the iteration step. The solution is then sought in the direction of the negative gradient, i.e.

$$x^{n+1} = x^n - k\partial_{x_i} g(x^n)$$  \hspace{1cm} (B.5)
where the parameter $k$ can be used to control the step size. The gradient $\partial_x g(x)$ is either evaluated analytically (if possible) or by means of \( (B.4) \). A converged solution found by this method will be a local minimum, however there is no guarantee that it is also a global one. This method is optimal, when no a priori knowledge of $g(x)$ is available.

### B.0.2. Method coordinate-wise iteration

This can also be done for each coordinate sequentially:

$$
  x_i^{n+1} = x_i^n - k\partial_x g(x^n)
$$

(B.6)

This yields control of the process for each coordinate, $x_i$, which is useful for cases where e.g. the optimal value of one or more coordinates is known a priori. The method has the same limitations as before, it cannot guarantee that the found minimum is global.

### B.0.3. The algorithm

The method developed in [17] is based upon the two above iterative methods. As stated these will not ensure automatic convergence to the global minimum. The method relies on some manual interaction for providing initial guesses for the coordinates and when the method converges to a sub-minimum. This concept is discussed in more detail in App. C on page 110, and the concrete method in [17].

### B.0.4. Example

In the present work a harmonic function is of special interest:

$$
  f(t, x_1, x_2, x_3, x_4) = x_1 + x_2 \sin(x_3 t + x_4)
$$

(B.7)

Applying \( (B.1) \) gives

$$
  g(x_1, x_2, x_3, x_4) = \|h(t) - x_1 + x_2 \sin(x_3 t + x_4)\|
$$

(B.8)

and with the least squares method

$$
  \|a\| = \sum_i a_i^2
$$

(B.9)

and for discretized time this becomes:

$$
  g(x_1, x_2, x_3, x_4) = \sum_{i=0}^{N} [h(t_i) - x_1 + x_2 \sin(x_3 t_i + x_4)]^2
$$

(B.10)
For this case the gradient can be evaluated analytically.

\[
\partial_x g(x) = \begin{cases}
\partial_{x_1} g(x) \\
\partial_{x_2} g(x) \\
\partial_{x_3} g(x) \\
\partial_{x_4} g(x)
\end{cases} = \begin{cases}
\sum (-2[h(t_i) - x_1 + x_2 \sin(x_3 t_i + x_4)]) \\
\sum (2[h(t_i) - x_1 + x_2 \sin(x_3 t_i + x_4)] \sin(x_3 t_i + x_4)) \\
\sum (2[h(t_i) - x_1 + x_2 \sin(x_3 t_i + x_4)] x_2 \cos(x_3 t_i + x_4)) \\
\sum (2[h(t_i) - x_1 + x_2 \sin(x_3 t_i + x_4)] x_2 \cos(x_3 t_i + x_4))
\end{cases}
\] (B.11)

By iterating (B.10) by means of (B.5) or (B.6) and using (B.11), a local minimum can be found.

### B.1. Application for 3D alignment

Consider the 3D marker coordinates

\[
x = \begin{cases}
x_i \\
y_i \\
z_i \\
1
\end{cases}
\] (B.12)

because affine transformations will later be used, the vector has a fourth element. We consider now that the camera is placed at the origin, pointing along the z-axis in the negative direction. Now the 3D geometry is first placed, in front of the camera with the right orientation. This can be realized with translation and rotation in three dimensions, i.e. by multiplication with the following matrices:

Rotation of \(\phi\) radians around the x-axis:

\[
M_x = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \phi & -\sin \phi & 0 \\
0 & \sin \phi & \cos \phi & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (B.13)

Rotation of \(\alpha\) radians around the y-axis:

\[
M_y = \begin{bmatrix}
\cos \alpha & 0 & \sin \alpha & 0 \\
0 & 1 & 0 & 0 \\
-\sin \alpha & 0 & \cos \alpha & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (B.14)
Rotation of $\beta$ radians around the x-axis:

$$M_z = \begin{bmatrix} \cos \beta & -\sin \beta & 0 & 0 \\ \sin \beta & \cos \beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (B.15)

Translation of $(x', y', z')$:

$$M_t = \begin{bmatrix} 1 & 0 & 0 & x' \\ 0 & 1 & 0 & y' \\ 0 & 0 & 1 & z' \\ 0 & 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (B.16)

Here the advantage of using an affine transformation is seen; translation can now be expressed by means of matrix multiplication. At this point the model is in the right position in front of the camera. A perspective transformation is of the form

$$M_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a_c & d_c \\ 0 & 0 & -1 & 0 \end{bmatrix}$$  \hspace{1cm} (B.17)

where $a_c$ and $d_c$ are free parameters [35]. A projection onto the image plane is simply done, by only considering the two fist coordinates, by multiplying with

$$M_{pr} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$  \hspace{1cm} (B.18)

and a scaling is realized by multiplying with

$$M_s = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$  \hspace{1cm} (B.19)

Lastly a quadratic lens correction, $xi(x, y)$ is made. Highly sophisticated lens models exist, but the simple quadratic expression has proved to be sufficient:

$$\xi(x, y) = 1 - a\sqrt{x^2 + y^2}$$  \hspace{1cm} (B.20)

The whole transformation from 3D-coordinates to 2D image coordinates can be thus be written:

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = \xi(x, y)M_sM_{pr}M_pM_tM_zM_yM_xx$$  \hspace{1cm} (B.21)

Now the general minimization problem solved for with the goal function of minimizing the discrepancy between the marker position on the images and the transformed 3D marker positions. With powerful symbolic algebra tools as e.g. Maple, the partial derivatives of (B.21) can actually be evaluated analytically which enables the algorithm to run accurately and efficiently.
C. Some words on Semi-automation

Some problems are best solved by a computer while some are best solved by the human brain, so much is evident. And then there are problems which are best solved in an interaction process. In this work several analysis procedures of this type is used, and some general remarks are therefore appropriate. Semi-automatic algorithms are a highly practical solution to complex problems. Here some aspects of these methods, their justification, and examples of their use for in the present work.

- **Interpretation of complex images**
  General and robust algorithms suitable for complex image analysis tasks and complex fitting tasks are mostly immensely complex and unfeasible to realize. Take e.g. the task of selecting the marker search regions (MSR). Most experiments will have some features which makes the images too unique for a general algorithm for this purpose to succeed. For example could the marker lie near an edge, whose inclusion in the MSR could make the method perform badly, or a piece of dirt or a pressure tap could be mistaken for a marker, etc. For a computer to solve these problems, it requires complex algorithms and a lot of programming work. For a human operator, however, these problems are very simple to solve. It is for instance very easy for the operator to identify the markers on the image, associate them with their 3D counterparts and choose a suitable size for the algorithm search regions, based on contrast, model geometry, edge effects etc. When the MSR has been determined, the computer can do the actual marker positioning with the mask correlation algorithm. A good piece of analysis software facilitates this work-sharing. It requires some manual work from the operator, but uses the input intelligently as to avoid excessive and repetitive operator tasks. With an appropriate graphical user interface (GUI) which makes quick testing, display and saving of settings easy, this is realized. When the human has found a set of MSRs which gives good results, the software can use different strategies for reuse of this information. For instance, since the profile does not move a lot from image to image, the MSR size is simply kept from on image to the next, while its position is centered around the found marker position of the previous image. Other strategies can be used, but this was not necessary in the present case.

- **Choice of initial values** A closely related problem is that of choosing initial values for iterative processes. The success of advanced iterative fitting with many free parameters is almost always depending on a reasonable choice of initial
values and tolerances. The human operator can narrow the parameter space for the initial values considerably. For instance, the the sinusoidal pressure signal, the order of magnitude for both amplitudes, phases and the frequencies can be estimated by the operator. Also tolerance and number of points used for the fit affects the quality. Typically the operator uses a typical set of measurement to find the setting producing a good fit, and then applies the same set to the following measurements. This method showed to be remarkably effective in practice, for e.g. the sinusoidal fitting of signals.

- **Result checking** It is very hard to supply the computer with general criteria for what constitutes acceptable solutions. These criteria has to be given in terms of error tolerances etc. and has to take an abundance of factors into account. A human however, needs only to throw a glance at the monitor to see if e.g. an image is properly aligned or a fit represents a data set properly. Any reducing of a data set, by characterizing it with a norm or a statistical moment leaves the possibility of overlooking some special feature of the raw signal. With a proper display of the fit results, the human eye is extremely good at assessing the quality of a fit, even at several data sets per second. When the fitting algorithm runs, the iteration process is displayed graphically as well, which makes it very easy to spot any major errors.
D. Table of PSP Probes
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<th>Matrix</th>
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Table D.1.: PSP probes (1)
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